

NAVAL WAR COLLEGE
Newport, R.I.

Chaos Theory:

The Essentials for Military Applications

by

Glenn E. James
Maj, USAF

A paper submitted to the Faculty of the Naval War College in partial satisfaction of the requirements of the Department of Advanced Research.

The contents of this paper reflect my own personal views and are not necessarily endorsed by the Naval War College, the Department of the Navy, or the Department of the Air Force.

Signature: Glenn E. James

21 February 1995

Paper directed by
John B. Hattendorf, Ph.D.
Director, Department of Advanced Research
and
The Ernest J. King Chair of Maritime History



Faculty Advisor
Stephen O. Fought, Ph.D.
Department of National Security Decision Making

Approved by:

DTIC QUALITY INSPECTED 8

STEPHEN O. FOUGHT

Faculty Advisor

21 FEB 95

Date

19950330 001

DISTRIBUTION STATEMENT A

Approved for public release;
Distribution Unlimited

REPORT DOCUMENTATION PAGE

1. Report Security Classification: UNCLASSIFIED			
2. Security Classification Authority: N/A			
3. Declassification/Downgrading Schedule: N/A			
4. Distribution/Availability of Report: UNLIMITED			
5. Name of Performing Organization: ADVANCED RESEARCH DEPARTMENT			
6. Office Symbol: 1C		7. Address: NAVAL WAR COLLEGE, 686 CUSHING RD., NEWPORT, RI 02841-5010	
8. Title (U): CHAOS THEORY: THE ESSENTIALS FOR MILITARY APPLICATIONS			
9. Personal Authors: JAMES, GLENN E.			
10. Type of Report: Final		11. Date of Report: 1995 FEBRUARY 21	
12. Page Count:			
13. Supplementary Notation: A paper submitted to the Faculty of the Naval War College in partial satisfaction of the requirements of the Department of Advanced Research. The contents of this paper reflect my own personal views and are not necessarily endorsed by the Naval War College, the Department of the Navy or the Department of the Air Force.			
14. Ten key words that relate to your paper: CHAOS, FRACTAL, RMA, NONLINEAR, DYNAMIC, CHAOS THEORY, MODEL			
15. Abstract: This report distills those issues of Chaos Theory essential for military decision makers, conducting a thorough review of Chaos applications in military affairs. Designed for courses at the various war colleges, the paper explains how to recognize Chaos, where to expect Chaos, and how to exploit the theories of Chaos and fractals. The essay includes a tutorial of nonlinear dynamics--assuming no math prerequisites--and an extensive bibliography. The universal features of chaotic system provide specific tools and methods to apply the theoretical results to problems in decision making and strategy formulation.			
16. Distribution / Availability of Abstract: A	Unclassified	Same As Rpt	DTIC Users
18. Abstract Security Classification: UNCLASSIFIED			
19. Name of Responsible Individual: Chairman, Department of Advanced Research			
20. Telephone: (401) 841-3304		21. Office Symbol: 1C	

DTIC	TAB	<input checked="" type="checkbox"/>
Unannounced		<input type="checkbox"/>
Justification		
By		
Distribution /		
Availability Codes		
Dist	Avail and/or Special	
A-1		

Acknowledgments

My thanks go first to Col John Warden, USAF, Commandant of the Air Command and Staff College (ACSC), who first asked me to consider this research topic. I gratefully acknowledge the Center for Naval Warfare Studies, who sponsored me under the Advanced Research Program, and Lt Col Griggs, in the Air Force Strategic Planning office, who also sponsored this work through the USAF Institute for National Security Studies at the U.S. Air Force Academy.

I owe a tremendous debt to my friend Maj Bruce DeBlois, USAF, at the School for Advanced Airpower Studies, Maxwell Air Force Base, Alabama, who fought hard to incorporate this research into the Air University curriculum, and to Lt Col Rita Springer and Lt Col Gus Liby, USAF, for arranging my presentation to the ACSC student body.

I especially appreciate the tremendous research and encouragement of my friend CDR Bill Millward, USN. He contributed an enormous volume of preliminary research and references that made this project possible in the short amount of time allotted.

My thanks also go to many individuals who patiently critiqued my seminars and drafts: the Naval War College faculty in Strategy and Force Planning; the Rational Decision Making faculty; Dr John Hanley, Program Director of the Strategic Studies Group, and the students in his Decision Theory Elective; Lt Col Lloyd Rowe, USAF; Maj Tom Tomaras, USAF. Finally, my deepest thanks to Dr Stephen Fought, my advisor, for his brave support and constant encouragement.

Preface

Before You Begin. . .

Before you start into this report, it may help you to relax and prepare to be patient.

Be Patient With the Material. . .

Chaos as a branch of mathematics is still extremely young. The first concrete results surfaced only 30 years ago. Enormous opportunities for new research remain unexplored. As of yet, not all the bodies of interested researchers know one another or exchange (or search for) information across disciplinary lines. This paper represents my effort to continue the published conversation on Chaos applications. I'm inviting you to eavesdrop because the issues are crucial to our profession.

Be Patient With the Essay. . .

Several officers got wind of my background in math, and as I left for the Naval War College, they asked me to consider how Chaos Theory influences the military profession. I saw the resources that were in use and I felt compelled to correct some serious errors. Many current publications overlook key results, they make fundamental

technical mistakes, or they scare the reader with the complexity of the issues. While the process documented in these papers is noteworthy—many sincere efforts were made under severe time constraints—we're overdue for a midcourse correction to prevent the errors from propagating any further.

My own Chaos research began in 1987 in my Ph.D. studies at Georgia Tech, where Professor Raj Roy introduced me to Chaos in lasers. Since then, I taught math for four years at the Air Force Academy, including three special topics courses on Fractals and Chaos. This past year, I made formal presentations to the ACSC student body and to two small seminars of Naval War College faculty. This paper grew out of those talks, subsequent questions, and my continuing research.

I've aimed this report at the broad population of students attending the various war colleges. I made the format conversational so I can talk **with** you, not **at** you, since this essay takes the place of what I might discuss in a more personal, seminar environment. I struggled to strike a useful balance, discussing more examples in some places, so I can reach this broad audience, cutting shorter in other places, to use your time more efficiently. I'm assuming a **minimal** technical background, appealing to an appendix only where absolutely necessary. I've also assembled a substantial bibliography of what I consider to be the best available references for the reader who's anxious for more.

Be Patient With Yourself. . .

Finally, relax. Chaos isn't hard to learn—it's hard to learn **quickly**. The important results are often abstract generalizations, but we can arrive at those conclusions via examples and demos that are not difficult to **visualize**. Allow yourself to **wonder**.

In his splendid text, *Fractals Everywhere*, Michael Barnsley warns:

There is a danger in reading further. You risk the loss of your childhood vision of clouds, forests, galaxies, leaves, feathers, . . . and much else besides. Never again will your interpretation of these things be quite the same.¹

I will warn you further of the risks of **not** reading further: you may fail to understand phenomena that are **essential** to decision makers, particularly in an era where the light-speed and high volume of feedback drive the dynamics of our physical and social systems into Chaos.

Contents

ACKNOWLEDGMENTS.....	ii
PREFACE	iii
CONTENTS	vi
LIST OF ILLUSTRATIONS	vii
EXECUTIVE SUMMARY	viii
INTRODUCTION	1
PART I	
WHAT IS CHAOS?.....	10
DEMONSTRATIONS	11
DEFINITIONS, TOOLS AND KEY RESULTS	35
PART II	
WHO NEEDS CHAOS THEORY?	69
EXPECT TO SEE CHAOS	70
HOW CAN WE USE THE RESULTS?.....	78
PART III	
WHAT NEXT?.....	142
SUGGESTIONS FOR FURTHER READING	143
FURTHER QUESTIONS TO RESEARCH.....	152
CONCLUSION.....	158
APPENDIX	160

List of Illustrations

Figure 1. Hypothetical Missile Test Data: What Went Wrong?	2
Figure 2. U.S. Aircraft Losses in Vietnam.....	2
Figure 3. Simple Pendulum, No Wobbles Allowed!	13
Figure 4. Stable and Unstable Equilibrium Points.....	16
Figure 5. Formation of Water Drops from a Spout.....	21
Figure 6. First-Return Maps for the Dripping Faucet.	25
Figure 7. Nightlight with Feedback.	28
Figure 8. Finding the Feigenbaum Constant in the Nightlight Experiment.....	31
Figure 9. Lorenz' Weather-in-a-Beaker.....	39
Figure 10. Chaotic Trajectories of the Iterated Logistic Map.....	45
Figure 11. Fractal Dimension: Always More Detail When You Zoom.....	49
Figure 12. Fractal Boundaries Between Basins of Attraction.....	50
Figure 13. Graphical Iteration of the Logistic Map.	52
Figure 14. Distribution of Logistic Map Dynamics.....	63
Figure 15. Chaotic Time Series for the Logistic Map What Comes Next?	95
Figure 16. Solid Lines Indicate Predicted Values. Dotted Lines Trace Actual Data.....	96
Figure 17. The Logistic Map, Stabilized with Chaos Control.	104
Figure 18. Periodic Changes in Force Levels in Two-Force Simulation.	114

Executive Summary

This report distills those issues of Chaos Theory essential to military decision makers. The new science of Chaos examines behavior that is characterized by erratic fluctuations, sensitivity to disturbances, and long-term unpredictability. This paper presents specific ways we can recognize and cope with this kind of behavior, in a wide range of military affairs.

Designed for courses at the various war colleges, the report makes three new contributions to the study of Chaos. First, it reviews the fundamentals of chaotic dynamics; the reader needs no extensive math prerequisites. Much more than a definition-based tutorial, the first part of the report builds the reader's intuition for Chaos and presents the essential consequences of the theoretical results. Second, the report surveys current military technologies that are prone to chaotic dynamics. Third, the universal properties of chaotic systems point to practical suggestions for applying Chaos results to strategic thinking and decision making. The power of Chaos comes from this universality: not just the vast number of chaotic systems, but the common types of behaviors and transitions that appear in completely unrelated systems. In particular, the results of Chaos Theory provide new information, new courses of action, and new expectations in the behavior of countless military systems. The practical applications of Chaos in military technology and strategic thought are so extensive that every military decision maker needs to be familiar with Chaos Theory's key results and insights.

to my Patient Family

Patricia Ann

Christine Marie

Phillip Andrew

who continue to follow me

bravely into Chaos

Introduction

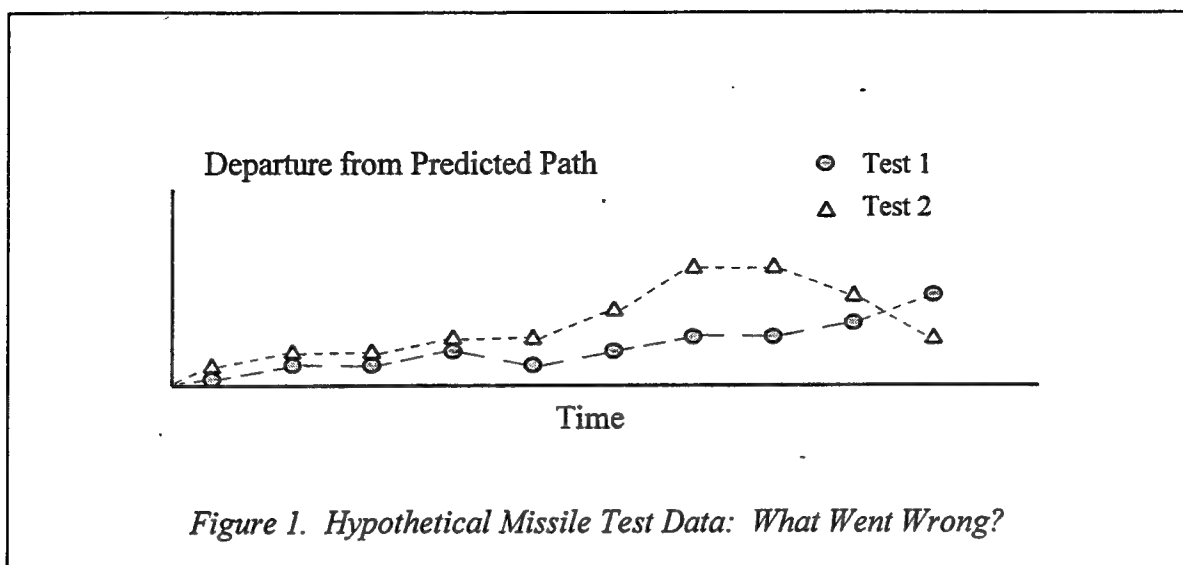
Welcome and Wonder

**Physicists, mathematicians, biologists, and astronomers
have created an alternative set of ideas.
Simple systems give rise to complex behavior.
Complex systems give rise to simple behavior.
And most important, the laws of complexity hold
universally, caring not at all for the details of a system's
constituent atoms.²**

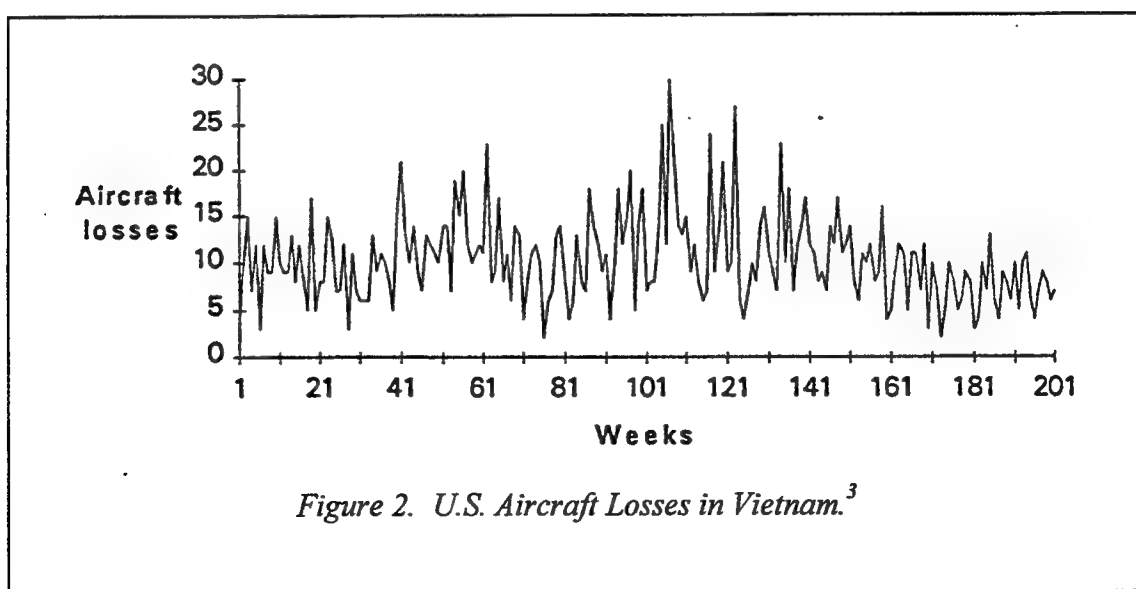
- James Gleick

Wake Up and Smell the Chaos

Your contractor for the operational tests of your new missile system just handed you the chart in Figure 1. He ran two tests, identical to six decimal places, but the system performance changed dramatically after a few time steps. He thinks there was a glitch in the missile's telemetry, or somebody made a scaling error when they synthesized the data. Could it be that the data is correct and your contractor is overlooking something critical to your system?



Your wargaming staff is trying to understand and model the time dependence of American aircraft losses in Vietnam. They look at the data in Figure 2 and quit. It's just a random scatter of information, right? No patterns, no structure, too many variables, too many interactions between participants, too large a role played by chance and human choice. No hope, right?



The results of the new science of Chaos Theory offer some intriguing answers to questions like these. Moreover, the theory has profound implications for the dynamics of an enormous array of military affairs. In fact, the applications of Chaos in military technology and strategic thought are so extensive that **every military decision maker** needs to be familiar with Chaos Theory's key results and insights.

Why Chaos with a Capital "C"?

Chaos, as discussed here, is **not** social disorder, anarchy, or general confusion. Before you read on, set aside your connotations of the societal (small "c") chaos reported on the evening news. Chaos **is** a relatively new discipline of mathematics with boundless applications; to highlight the difference, I'll keep Chaos capitalized throughout the essay. Chaos Theory describes a specific range of irregular behaviors in systems that move or change. What is a system? To define a system, we only need two things: a collection of elements—components, players or variables—along with a set of rules for how those elements change—formulas, equations, recipes, or instructions.

The term Chaos was first coined less than 30 years ago—that's a **hot** topic for math! James Yorke characterized as **chaotic** the apparently unpredictable behavior displayed by fluid flow in rivers, oceans and clouds.⁴ Now, artificial systems can move and react fast enough to generate similar, erratic behavior, dynamics that were not possible before the advent of recent technologies. **Many** military systems exhibit Chaos, so we need to know how to recognize and manage these dynamics. Moreover, the

universality of many features of Chaos gives us opportunities to exploit these unique behaviors. Learn what to expect. This is not a fleeting fad: **real systems** really behave this way.

What's New in This Essay?

There are plenty of Chaos tutorials already, in various disciplines, but there are three main deficiencies in the available resources:

1. Many tutorials require an extensive background in math analysis.
2. Many works do not focus on useful CONSEQUENCES of Chaos Theory. They simply offer a smorgasbord of vocabulary.
3. Many contain major technical flaws that dilute their potential application or mislead the reader.

So, the immediate need is two-fold: we need a bridge to connect us with the basics of Chaos Theory, and we need in-depth demonstrations of its applications. This essay strives to serve both these needs.

Who Cares?

Even if Chaos can help military **analysts**, why should **everyone** be exposed to the theory? After all, there's a balance here: you don't need to know quantum physics to

operate a laser printer, right? However, you're about to see that Chaos occurs in virtually every aspect of military affairs. The 1991 DOD Critical Technologies Plan, for instance, sets priorities for research spending, prioritizing the following technologies based on their potential to reinforce the superiority of US military weapon systems:⁵

1. semiconductor materials & microelectronics circuits
2. software engineering
3. high performance computing
4. machine intelligence and robotics
5. simulation and modeling
6. photonics
7. sensitive radar
8. passive sensors
9. signal and image processing
10. signature control
11. weapon system environment
12. data fusion
13. computational fluid dynamics
14. air breathing propulsion
15. pulsed power
16. hypervelocity projectiles and propulsion
17. high energy density materials
18. composite materials
19. superconductivity
20. biotechnology
21. flexible manufacturing

Every one of these technologies overlaps with fundamental results from Chaos Theory!

In particular, the **chaotic dynamics possible** in many of these systems arise due to the **presence of feedback** in the systems; other sources of Chaos are discussed elsewhere in this report. As you continue reading, you'll see that the fundamentals of Chaos are as important to military systems as Newton's laws of motion are to classical mechanics.

Numerous systems tend to behave chaotically, and if you don't understand Chaos, you won't understand a great deal of the changes happening around you. Look again at Figure 2. Not too long ago, if we had measured output like Figure 2, in any scenario, we would have packed up and gone home. However, it's not "noise" at all. Chaos Theory helps us know when erratic output, like that in the figure, may actually be generated by deterministic (**non-random**) processes. In addition, the theory provides an astounding array of tools which:

- make **short-term predictions** of the next few terms in a sequence;
- **predict long-term trends** in data;
- **estimate how many variables** drive the dynamics of a system;
- **control dynamics** that are otherwise erratic and unpredictable!

Moreover, this analysis is often possible **without any prior knowledge of an underlying model or set of equations.**

Applied Chaos Theory already has a growing community of its own, and the majority of military decision makers are not yet part of this group. For example, the Office of Naval Research (ONR) leads DOD research into Chaos applications in engineering design, but more **military leaders** need to be involved and aware of this progress. Beyond the countless technical applications, many of which readily translate to commercial activities, we must concern ourselves with strategic questions and technical

applications that are unique to the warfighting profession. Chaos Theory brings to the table practical tools that address **many** of these issues.

Why Now?

As long as there has been weather, there have been chaotic dynamics—we're only **recognizing** it now for the first time. Previous scientists, like Poincaré in the late 1800s, had some inklings of the existence of Chaos, but the theory and the necessary computational tools have only recently matured enough to study chaotic dynamics. Edward Lorenz, in 1963, made his first observations of Chaos quite by accident, when he attempted simulations which were only possible with the advent of "large" computers. Currently, high-speed communication, electronics and transportation bring new conduits for feedback, driving more systems into Chaos. Consider, for instance, the weeks required to cross the Atlantic to bring news of the American Revolution to Britain. Now, CNN brings live combat updates from Baghdad.

Until now, observations of the irregular dynamics that often arise in rapidly fluctuating systems have been thrown away. Unless we **train** decision makers to look for specific dynamics and symptoms of the imminent onset of drastic transitions in behavior, erratic data sets will continue to be discarded or explained away.⁶

Clear Objectives

As I suggested in the Preface, Chaos Theory is not difficult to learn—it's difficult to learn **quickly**. Am I violating this premise by trying to condense the essentials of Chaos into this single report? I hope not. I'm trying to build a bridge and sketch a coarse map. The bridge spans the gap that separates physical scientists on one side—including analysts in math, physics, and electrical engineering—and humanists on the other—experts in psychology, history, sociology and military science. My bibliographical map points the way to more specific references, on issues that interest smaller segments of the broad audience I hope to reach with this essay.

My intent here is to teach you just enough to be dangerous, to convince you that Chaos happens all around you. The results of Chaos Theory can improve your decision making and add new perspectives to your creative thought. I will also show you enough examples and applications that you won't be able to help yourself: you'll start to notice chaotic dynamics wherever you are. Eventually, I hope you'll grasp the key results and use them to your advantage in your particular discipline. Finally, I aim to infect you, irreversibly: with a new perspective on motion and change, with an insatiable curiosity about Chaos, and with adequate tools and references to continue the deeper study which is essential to fully understanding the fundamentals of Chaos.

Here's the plan. In Chapter One, we start with Chaos you can demonstrate at home, so the skeptics will believe Chaos is more than a metaphor, and so we can all visualize and discuss important issues from a common set of experiences. I don't want

you to mistakenly believe you need access to high technology circuits and lasers to concern yourself with Chaos—quite the contrary. Then we'll add some detail in Chapter Two, complementing our intuition with better definitions. In Chapter Three, we take a look at the pervasiveness of Chaos in military systems. Next, Chapter Four serves up **practical means** for applying Chaos Theory to military operations and strategic thinking. Most of the discussions proceed from specific to general in order to lend a broad perspective of how Chaos gives **new information, new options for action, and new expectations of the dynamics** possible in military systems.

In the end, I hope you will learn to:

- RECOGNIZE CHAOS when you encounter it;
- EXPECT CHAOS in your field, your organization, and your experiments;
- EXPLOIT CHAOS in your decision making and creative thought.

PART I

What IS Chaos?

**Somehow the wondrous promise of the earth
is that there are things beautiful in it,
things wondrous and alluring,
and by virtue of your trade
you want to understand them.⁷**

- Mitchell J. Feigenbaum

ONE

Demonstrations

**The disorderly behavior of simple systems...generated complexity:
richly organized patterns,
sometimes stable and sometimes unstable,
sometimes finite and sometimes infinite,
but always with the fascination of living things.
That was why scientists played with toys.⁸**

- James Gleick

DEFINITELY Try This at Home!!!

The simple demonstrations in this chapter offer visualizations of a wide range of chaotic dynamics. You will also get a good introduction to the methods and tools available to observe, measure and analyze these dynamics. My main goal is to build your **intuition** for what Chaos looks like.

For any skeptical readers, these examples represent the first exhibits of the extensive evidence I will produce to demonstrate how prevalent chaotic dynamics are. For all readers, this chapter outlines common examples which provide a useful

context—in subsequent chapters (and in your own deliberations)—for discussing definitions, tools, and key results, and applications. I chose to begin with demonstrations you set up at **home**, so you don't get the impression you need access to high technology to observe Chaos. Quite the contrary, I hope you will be astounded at how Chaos arises in the simplest physical systems. You may also find that even this brief exposure to chaotic dynamics will spark your own imagination about the systems where Chaos may arise in your own areas of interest. A little later, after a more complete description of the vocabulary and tools of Chaos (Chapter Two), we will review the **military** systems where you should expect to see Chaos (Chapter Three).

Remember: as you work through each example, you will gradually come to expect and recognize Chaos in **any** system that changes or moves. As a general plan for each demo that follows, I will:

1. Describe the physical system and clearly answer:

What is my system?

What am I measuring?

2. Preview the significance of the particular system:

Why do we care about **this** demo?

What does it let us see?

Does it relate to any military systems?

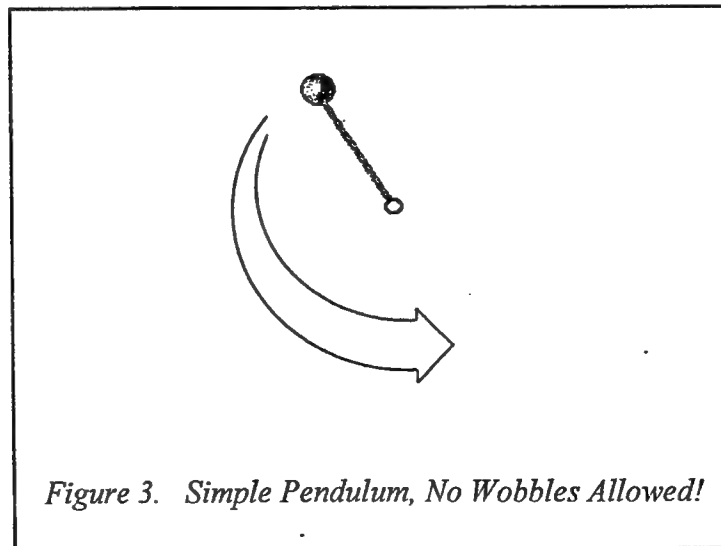
3. Discuss the significant dynamics and transitions.

4. Highlight those results and characteristics **common** to many chaotic systems.

The answers in Item 1 are crucial. The confusion in many discussions about Chaos can be traced to failures to identify either a well-defined system or a set of measurements. Likewise, to understand the appropriate ways to **apply** the insights of Chaos, we need to use its terminology with some care. With this priority in mind, the discussion of each demo will give you a first peek at the Chaos vocabulary which Chapter Two presents in greater detail.

Yo-Yo —Warm Ups with a Single Pendulum

Before we exercise our imaginations with chaotic dynamics that may be entirely new, let's first "stretch out" by examining the detailed behavior of a yo-yo. The simplicity of this example makes it easy to **visualize** and to **reconstruct** in your home or office; it gives us an indication of good questions to ask when we observe other systems.



As a hint of things to come, an extraordinary number of complicated physical systems behave just like a pendulum, or like several pendula that are linked together. Picture, for instance a mooring buoy whose base is fixed to the sea floor, but whose float on the surface is unconstrained. Much of the buoy's motion can be modeled as an upside-down pendulum.⁹ Nevertheless, while we will **eventually** get a lot of mileage out of the motion of a single pendulum, just think of this as warm-ups.

What is the system, exactly? A fixed mass, suspended at the end of a fixed length of rigid wire (let's not allow our strings to bend here; don't let your yo-yo roll up and down), swings in only two dimensions (left and right swings only, no additional motion). The wire is fixed at a single point in space, but let's assume there is no "ceiling" so the yo-yo is allowed to swing up over its apex and around to the other side (Figure 3). Please notice that, as we define the system, we must clearly state our **assumptions** about its components and its behavior.

What can we observe and measure in this system? Fortunately, in this example, we only need two pieces of information to completely describe the physical "state" of the system: **position** and **velocity**. The pendulum's position is measured in degrees; its velocity is measured in degrees per second. These two observable quantities are the only two independent variables in the system, sometimes referred to as the **degrees of freedom** or the **phase variables** for this system. A system's phase variables are those time-dependent quantities that determine its state at a given time. Notice that,

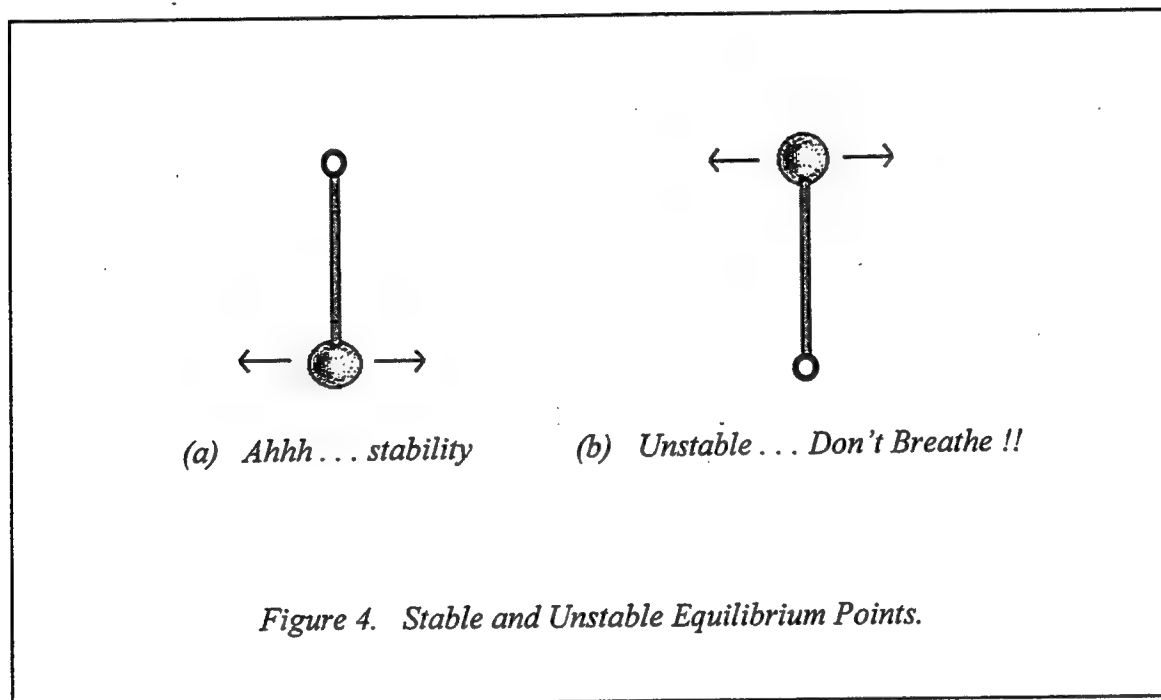
even though the yo-yo swings in a curve that sits flat in a two-dimensional plane, we only need one variable to describe the yo-yo's position in space. Therefore, the yo-yo has only **one degree of freedom** in its angular position.

So, what can this yo-yo do? Let's pretend, at first, that the yo-yo experiences **no** friction, drag, or resistance of any kind. This **ideal** pendulum exhibits a rich variety of behavior. If we start it at "the bottom," where both position and velocity are zero, it stays there. Any state that has this property—not changing or moving when undisturbed—is called an **equilibrium, steady state, or fixed point** for the system. If we displace the yo-yo a few degrees to either side, and just let it go, it swings back and forth **periodically**, with the same undying amplitude, forever. In this ideal system, we can also carefully balance the pendulum at the **top** of its swing, and it will stay put forever. This state, with position 180 degrees and velocity zero, is another **equilibrium point**.

Does this ideal pendulum display any other dynamics? Perhaps just one more: we can impart enough **initial velocity** to the pendulum so it swings upwards over its apex and continues to wrap around its axle forever. This completes the list of possible dynamics for the ideal pendulum, and it completes your first exposure to some important terms used to describe all **dynamical systems**.

Now let's get back to reality and add some resistance to the system, where the yo-yo experiences "damping" due to friction and drag. This real pendulum still has the same two equilibrium points: the precise top and bottom of its swing, with zero velocity. A new feature we can discuss, though, is the **stability** of these **fixed points**. If we disturb any pendulum as it hangs at rest, it eventually slows its swing and returns to this low

equilibrium. Any such fixed point, where small disturbances “die out,” and the system always returns to its original state, is called a **stable fixed point** (Figure 4a). On the other hand, at the top position of 180 degrees, any perturbation to the right or left sends the yo-yo into a brisk downswing that eventually diminishes until the yo-yo hangs at rest. When a system tends to depart from a fixed point with any minuscule disturbance, we call it an **unstable fixed point** (Figure 4b).



We should also note a few other issues, concerning the yo-yo's motion, which will arise when we study more complicated systems. First is the observation that the pendulum (with friction) displays both **transient** and **limit dynamics**. The yo-yo's **transient dynamics** are all the swings which eventually damp out due to resistance in the environment. After all the transients die out, the system reaches its **limit dynamics**, which in this case is a single state: the lower fixed point with zero position and velocity.

It seems we may be reaching the point where we've exhausted the possible dynamics in this simple pendulum system. After all, even though this is a harmless way to introduce the vocabulary of fixed points, dynamics, transience and stability, there is only so much a yo-yo can do. Right?

When the system remains undisturbed, the answer is a resounding YES! The reason: the motion of a simple pendulum, unforced, is a **linear system** whose solutions are all known. In particular, the equation of motion, for the position y , comes from Newton's familiar relation, force equals mass times acceleration:

$$m y'' + c y' + k y = 0 , \quad (1)$$

where m is the pendulum mass, c is a measure of friction in the system, and k includes measures of gravity and string length.

Now, the swinging motion we **observe** appears to be anything but linear: a pendulum swings in a curve through space, not a straight line, and the functions that describe oscillations like these are wavy sines and cosines. However, the **equation of motion**—like the system itself—is called **linear** because the equation consists of only **linear operations**: addition, multiplication by constants, and differentiation. When the pendulum experiences no external forces, the resulting **homogeneous** equation shows a zero on the right hand side of Equation (1). What's the significance of recognizing a linear, homogeneous system? **All** the solutions are **known**; all possible behaviors are known and predictable.

To add the last essential layer of reality, and to generate some interesting motion in the pendulum system, envision a playground swing. Once you get yourself started swinging, how do you get yourself to swing much higher? You add a relatively small external force to the system: you kick your legs, and lean forward and back, in a manner carefully timed with the larger motion of the swing itself. By pumping your legs like this, you add a periodic force to the right side of Equation (1), and you resonate and amplify a natural frequency of the large swing.

This addition of an external kick, or forcing function, to the pendulum system can induce interesting new dynamics. Be aware that, especially if you are pushing someone else on the swing, you can control three different features of the perturbation: where you push, how hard, and how often. The system may respond to the external forcing in many different ways. It may resonate with one of its natural frequencies (you may have seen the movie of the Tacoma Narrows bridge, destroyed by the violent oscillations induced by resonance with wind gusts). In another instance, the swing may behave quite unpredictably if you lose your concentration and push the chains instead of the swing. You may momentarily bring the entire system to a halt, cause intermittent lurches in the swing, or you may get very regular motion for a long time, only occasionally interrupted by off-cycle bumps or jostles.

The playground swing, as a system, is just like the simple pendulum. However, when you “kick” it occasionally you begin to observe departures from predictable behavior. This irregular sort of behavior, characteristic of a kicked pendulum, is one of the many traits of Chaos: behavior that is **not periodic, apparently random**, where the

system response is still recurrent (the pendulum still swings back and forth) but **no longer in a predictable way**. Because of the rich dynamics possible in such a simple physical system, James Gleick correctly asserts that physicists could not truly understand turbulence or complexity until they understood pendulums. Chaos Theory unites the study of different systems, so the dynamics of swings and springs broaden to bring new insights to high technologies from lasers to superconducting Josephson junctions, control surfaces in aircraft and ships, chemical reactions, the beating heart, and brainwave activity.¹⁰

As this essay continues, we will see more detailed connections between the behavior of yo-yo's and other more complicated systems. For now, let's move on to our second home demonstration of Chaos, introduce some additional vocabulary, and continue to build our intuition for what we should expect to see in a chaotic system.

The Dripping Faucet

Our second home demo can be done at the kitchen sink, or with any fluid spout where you can control the fluid flow and observe individual drips. This demo mimics an original experiment by Robert Shaw and Peter Scott, at the University of California Santa Cruz.¹¹ This is a wonderful illustration of several features of Chaos, particularly the **transitions** between various dynamics, which are common to many systems. The results are so astounding, you may want to bring your reading to the sink right now and experiment as you read along. Otherwise, you may not believe what you read.

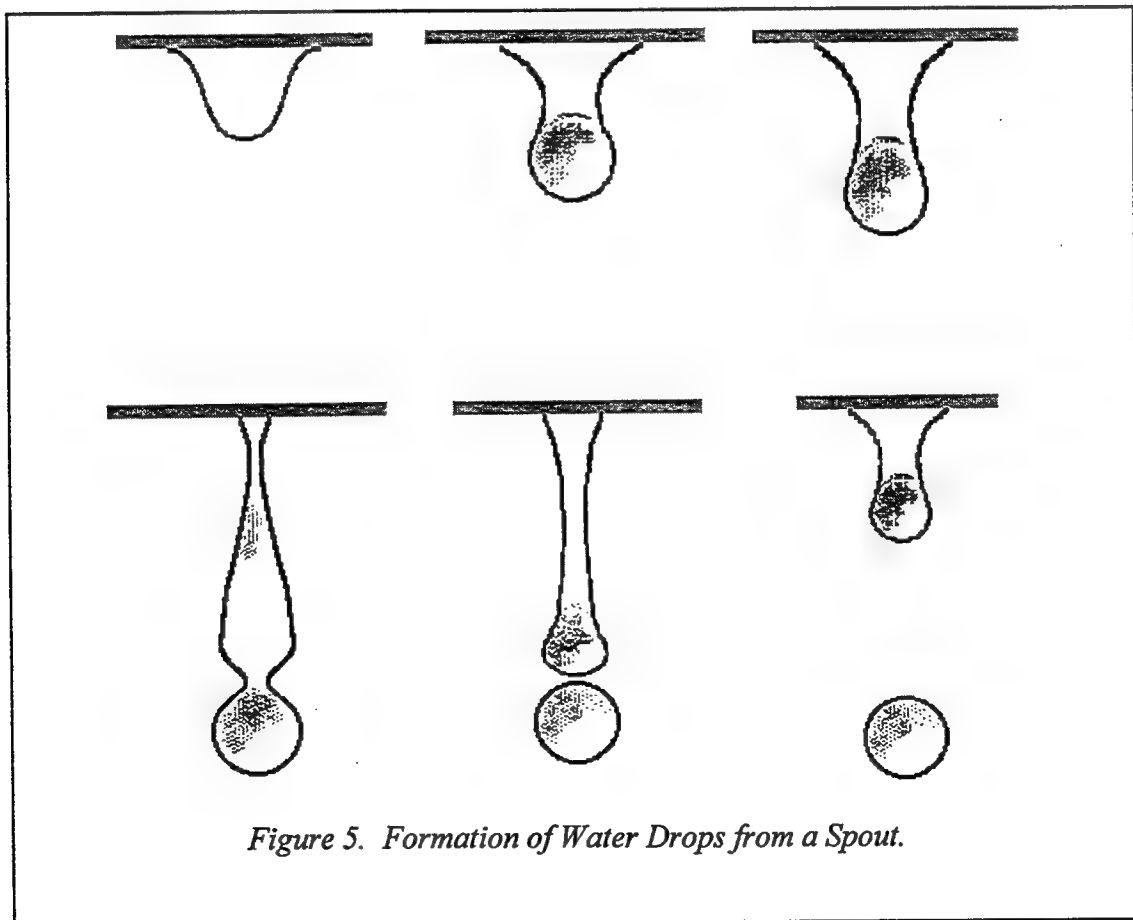
What's the system? To recreate the Santa Cruz experiment, we need a faucet handle or spigot that can be set at a slow flow rate and then left alone so we can observe drops emerging for a few minutes. We need enough water available so the flow continues without interruption (which is only a problem if you're using an iced tea cooler, or a mustard squeeze-bottle, as your controllable spout). Finally, you need some means to observe the timing between drops. You don't need a stopwatch, exactly, but you do need either a clear view of the drops, or you need the drops to land on some surface that resounds loudly enough for you to observe patterns and rhythms as the drops fall. Fortunately, we need no assumptions about the water quality, or any details about the size or material of the spout. We just need drops.

What can we observe and measure in this system? We want to have a clear view of the drops forming; this will give us some intuition for **why** the flow makes transitions between different kinds of behavior. We want to **measure the time intervals** between drops. Shaw and Scott did this very precisely with a laser beam and computer. For us, it will suffice to watch or listen as the drops land.

What's the significance? Because of the difficulties modeling any fluid, there is absolutely **no hope of simulating** even a single drop forming and dropping from a faucet. However, by measuring only one simple feature of the flow, the **time between events**, we can still understand many characteristics of the system dynamics. We'll observe, for

example, specific **transitions** between behaviors, transitions that are common to many chaotic systems. We'll also gain some useful metaphors that are consistent with our intuitions of human behavior, but much more important, we'll learn some **very specific things to expect** in chaotic systems, even when we can't model their dynamics.

So, what kind of things can a sequence of water drops do? If the spigot is barely open, and the flow extremely slow, you should observe a slow, regular pattern of drips. Leave the faucet alone, and the steady, aggravating, **periodic** rhythm will continue, far into the night. This pattern represents **steady state, periodic** output for this system.



Increase the flow ever so slightly, and the drips are still periodic, but the **time interval** between drips **decreases**, that is, the **frequency increases**. At the other extreme of its behavior, with the flow rate turned much higher, the water will come out as a steady, unbroken stream. No real excitement, yet.

The big question is: what happens **in between** these two extreme behaviors? How does the flow make its transition from periodic drips to a steady stream? We'll move step by step through the transitions in this system. Low flow rates will generate slow, regular drips. Increased flow will produce regular drips with new patterns. After a certain point, the drop dynamics will prevent the faucet from dripping regularly, and we'll see evidence of Chaos.

Here's how to proceed with the experiment. Start with a slow steady, dripping faucet. Watch, for a moment, how the drops form. A single drop sticks to the end of the spout and begins to fill with water, like the elastic skin of a balloon (Figure 5). Eventually, the drop grows large enough to overcome its surface tension; it breaks off and falls. The water left on the spout first springs back and recovers, then it begins to fill up to form the next drop. We will see that the time it takes the second drop to recover is a critical feature of the system.

Now, VERY gradually, increase the flow rate. For a while, you'll still see (or hear) periodic dripping, while the frequency continues to increase. However, before too long—and before the flow forms a solid stream—you will observe a different dripping pattern: an irregular pattern of rapid dripping interspersed with larger splats of various

sizes, all falling at erratic, unpredictable time intervals. What causes the new behavior? The drops are beginning to form so quickly that a waiting drop does **not** have time to spring back and completely recover before it fills with water and breaks off. This is **chaotic** flow.

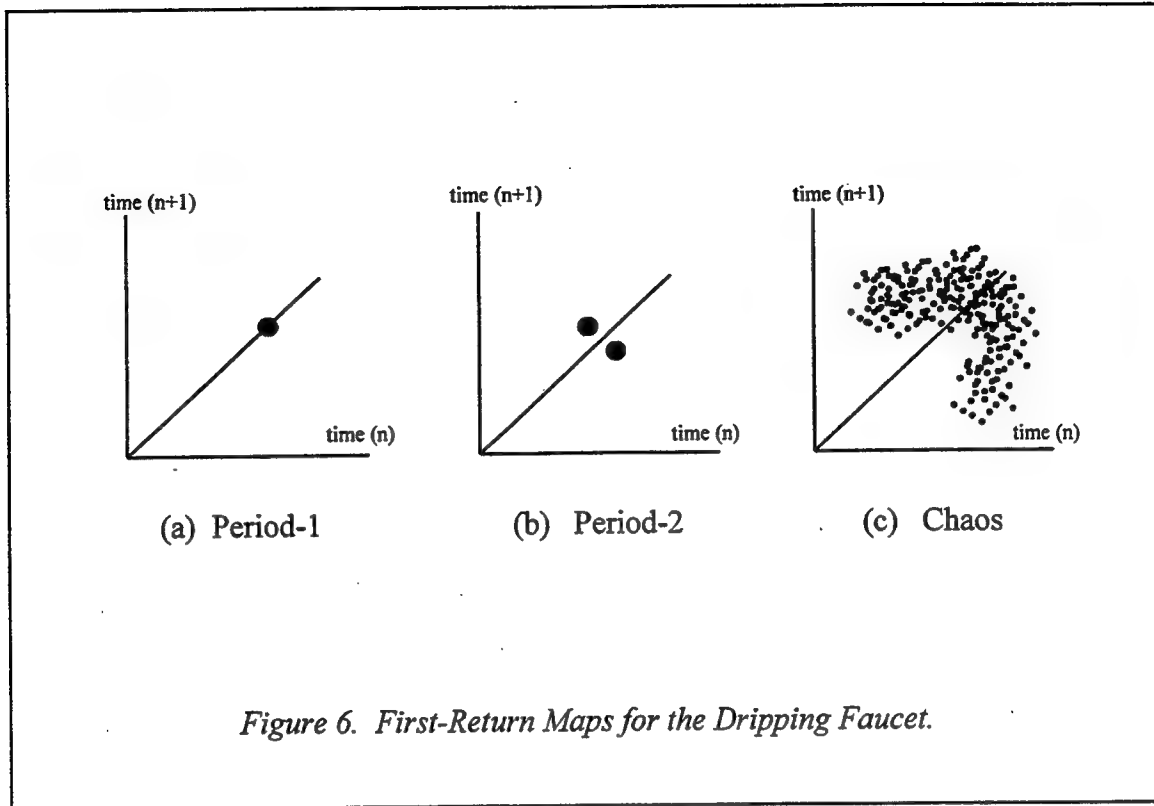
This deceptively simple demo is essential to our intuition of Chaos, for several reasons. First, despite the nasty fluid physics that's **impossible to model** in detail, we are able to make simple measurements of time intervals and learn a great deal about the system dynamics. We learn, in this experiment, that we need not dismiss as intractable the analysis of a system that seems to be too large or has "too many variables" or "too many degrees of freedom." You can surely imagine quite a few military systems with these imposing properties, starting with a conventional battlefield. The water drops give us hope: by isolating and controlling **one key parameter**, and making one straightforward measurement, we can still come to understand, and perhaps manipulate, a very complicated system.

The second common feature of Chaos illustrated by the dripping faucet is the presence of this **control parameter**—in our case, the flow rate, controlled by your spigot. Think of a control parameter as a single knob that allows you to regulate the amount of energy in the system. Not only does this energy control provide a means to dictate the dynamics of the dripping faucet, but the **transitions between various behaviors are identical** in countless, seemingly unrelated physical systems. In the faucet, for instance, low flow generates periodic output; an increase in flow leads to higher-period behavior; even higher flow—more energy in the system—allows chaotic dynamics. Moreover, the

Chaos appears when the system has insufficient time to relax and recover before the next “event” occurs. These same transitions take place in mechanical systems, electrical systems, optical systems, and biological systems. Even more incredible, the transitions to more complicated behavior can occur at predictable **parameter** values (“knob” settings), a result discussed in the demonstration that follows. The **critical conclusion** is that our knowledge of chaotic systems teaches us:

1. to expect specific behaviors in a system that displays periodic behavior;
2. to expect to see higher-periods and Chaos with more energy input;
3. to forecast, in some cases, parameter values that permit these transitions!

A third common characteristic of chaotic systems highlighted here is the fact that the system dynamics are revealed by observing **time intervals between events**. The physical event—droplet formation and breakoff—is impossible to simulate, so we avoid taking difficult measurements like drop diameter, drop mass or velocity. Instead, we note the length of time between events, and, if we can measure this time accurately, we are able to construct a **return map**, or **first-return map** that clearly indicates the various patterns of behavior (Figure 6).



On the x-axis, a return map plots the time difference between, say, drops 1 and 2, versus the y-axis time difference between drops 2 and 3. When the flow is slow and periodic, the time intervals are regular, so the time between the first drops is equal to the time between the next pair of drops. On the plot, that means we're plotting x-values and y-values that are always equal, so we see a single dot on the plot (Figure 6a). So, if we ever observe a return map where all the data fall on a single point, we can conclude our system is behaving periodically.

If we consider our time-difference measurement a record of the state of our system, then any **limit** behavior summarized on the return map represents an **attractor** for the system. An **attractor** is a collection of states that a system "settles" into after its

transient dynamics die out. For the periodic flow, the attractor is a single point on the return map.

The next transition in drop dynamics was reported by Shaw and Scott but is fairly difficult to perceive in our home experiment. At a specific range of flow rates, before the onset of Chaos, they observed a rapid string of drips that fell off in close pairs. The flow showed a different periodicity with one short time step followed by a longer time step: drip-drip drip-drip drip-drip. . . . They confirmed the existence of this change in periodicity by using a simple model of their system, but its presence was clear on the return map (Figure 6b).¹² In this case, we say the sequence of drops has **period-2**, we say the system has undergone a **period doubling**, and the attractor is the set of two points on the plot. For the record, this system (like many others) experiences additional period doublings to **period-4**, **period-8**, etc., before the onset of Chaos. These transitions, however, can be difficult to detect without sensitive lab equipment.

Finally, **chaotic flow** generates time intervals with no periodicity and no apparent pattern. However, the chaotic return map does **not** simply fill **all** the available space with a random smear of points (Figure 6c). There is some rough boundary confining the chaotic points, even though they appear to fill the region in an erratic, unpredictable way. What is most astonishing is that this smear of points is amazingly reproducible. That is, we could run the experiment anywhere, with virtually any water source, and a very similar pattern of points would appear on the return map for chaotic flow. The structure of the collection of points is due to the dynamics driving water drops in general, not due to the specific experimental machinery.

The water drop experiment offers additional hope for how we might **control a chaotic system**. (This is easiest to demonstrate if you use a portable water spout, like an empty mustard bottle, but may work well if your kitchen spout is sufficiently flexible.) Set the spout so you have flow that remains chaotic. Then try the following: jiggle the spout in some regular, periodic way. You might bounce your mustard bottle up and down, or simply tap the end of your kitchen faucet with a regular beat. You should be able to find the right strength and frequency to perturb your system and get it to change from Chaos back to periodic drips, a periodicity that will match the beat of your tapping. This is not very different from kicking your legs on the swing. However, in this case, we are **starting** with a chaotic system where we can apply a relatively small disturbance to **force** the system to return to more stable periodic behavior.

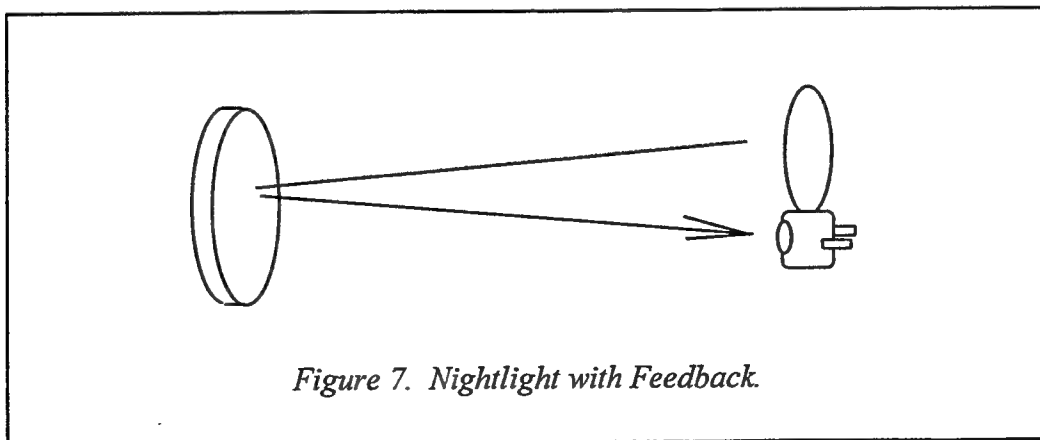
Later, the essay will discuss more details concerning **Chaos control** that has been successfully demonstrated in both theory and practice. We will also consider issues of when we might prefer Chaos to be present or not present in a given system. At this point, it is interesting to notice, too, that the **perturbation** of the dripping faucet can drive a chaotic system into stable (periodic) behavior, while our previous perturbation of the park swing forced it to go **from stable periodicity into Chaos!**

At this point, we've introduced two chaotic systems whose dynamics will lend some insight to the behavior of more complicated military systems. The first was mechanical, the second, fluid. Our next demo involves some simple (inexpensive!) electro-optics you can pick up at any hardware store.

Nightlight

I stumbled onto this demonstration quite accidentally, just a few weeks ago. I went to plug in a small nightlight in our bathroom—one of those automatic lights, about the size of your palm, that turns on automatically when the room is dark. I plugged it into the socket; the room was dark. I noticed that, just before I pulled my hand away from the nightlight, it flickered rapidly. I put my hand near the light again and I saw the same flicker. What interesting dynamics are hiding in this system?

What's the system? To reconstruct this system we need a light source of any kind that includes an automatic sensor that cuts off the electric current when it senses light (Figure 7). We also need a dark room (as opposed to a darkroom) and a mirror, small enough so we can move it around near the light, and supported in a stand so we can let it go, in order to observe the light. Now, set the mirror so it reflects light from the bulb back onto the sensor. By adjusting the mirror's distance from the sensor, we can **vary the delay of feedback** in the system.



What are we observing and measuring? When the mirror is close enough to the nightlight, about four to twelve inches, you should see it flicker. What's going on? Quite simply, the sensor is doing its best to fulfill its mission under unusual circumstances. Initially, the room is dark, so the sensor turns its light on. You've put your mirror in place, though, so as soon as the light turns on, the sensor picks up the reflected light and correctly decides to shut off. Oh dear, the room is dark again: time to turn on, and so on. The sensor detects and responds very quickly, so we see the nightlight flicker vigorously.

What exactly should you observe in this system? Like the dripping faucet, the output to measure here is the **frequency** of the flickering, the time difference between events. We would probably learn even more by also monitoring the light's intensity, but this requires fancier equipment than most of us keep around the house.

What transitions should we expect? To see the range of dynamics possible in this system, start with the mirror far from the sensor, about a yard or so away. Slowly draw the mirror closer to the sensor. The first change you'll see is a noticeable dimming in the light. Honestly, I don't know yet whether this is a simple change in the light's output or a fluctuation whose frequency exceeds our visual resolution. Do your best to locate the **farthest** point from the light where the dimming begins. Let's label this distance, d_0 . You may find that d_0 is up to a foot or two away from the light.

As you move the mirror even closer, the next change you'll probably see is the first sign of flickering. Once again, try to mark d_1 , the farthest place where the flicker is noticeable. As you continue to move the mirror toward the sensor, you will see various

ranges of distances where the flickering displays other periodicities, and you ought to see at least one region where the reflected feedback drives the system into Chaos: irregular bursts of brightness and flickering. Mark the distances, as well as you're able, where you see transitions: d_2 , d_3 , etc. If you don't observe any Chaos, how might you alter your system? There are several accessible control variables: try a different (cleaner?) mirror; change your reflection angle (are you hitting the sensor efficiently?); use a brighter light bulb.

What's the significance? The dynamics exhibited by the nightlight system highlight several critical insights that will help us apply the general results of Chaos Theory to other systems. The first new insight comes from the dynamics we can generate by imposing **feedback** on a system. Of course, the use of feedback itself is not new, but the **output** we observe in the nightlight provides a new understanding of the dynamics that control theorists have been wrestling for decades.

The nightlight demo also offers **practical** new approaches to study and control a system whose output sometimes fluctuates. In particular, once I observed periodic behavior in the system (accidental though it was) I knew to expect several ranges of periodicity and Chaos if I varied one of the control parameters available to me. My experience with Chaos gave me very specific behaviors to **expect**, in addition to obvious suggestions of ways to **control the dynamics**. Moreover, I had some idea of the kinds of **dynamics to expect without knowing anything about the internal workings of the system!**

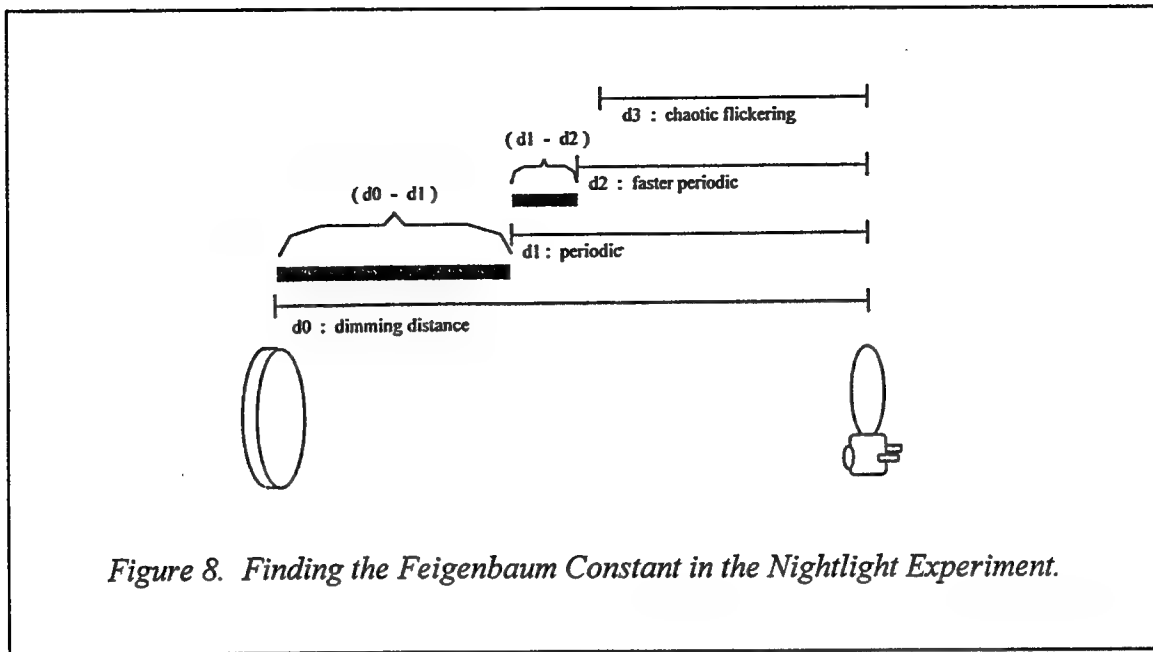


Figure 8. Finding the Feigenbaum Constant in the Nightlight Experiment.

This universality of chaotic dynamics underscores the power of understanding the basic results of Chaos Theory. Certainly, not every system in the world is capable of generating Chaos, but in many systems we can control and analyze a system with no need for a model. Here are two simple examples of the kind of analysis that's possible even **without a model**. For this analysis we only need the list of distances (d_0 , d_1 , etc.) where we noted transitions in system behavior. First of all, we know that the signal in our demo, the light from the bulb, is traveling at a known constant, $c = 3.0 \times 10^8$ meters/sec. Therefore, we can quickly assemble a list of important **time constants** for this system by dividing each of our distances by the speed of light, c . These time constants directly affect important transitions in the light's output; we know we can alter the system's behavior by applying disturbances that are faster or slower than these key time delays. Other time constants we might consider include the frequency of the electric current, and the frequency (color) of the light.

A second numerical result gives us some hope of **predicting the parameter values** where transitions in dynamics should occur. Dr Mitchell Feigenbaum, of Los Alamos National Laboratory, New Mexico, discovered that many chaotic systems undergo transitions at predictable ranges of their parameter settings. In particular, he compared the **ratio** of differences between key parameter values, which for us translates into calculating a simple ratio:

$$(d_0 - d_1) / (d_1 - d_2) . \quad (2)$$

He discovered that this ratio is a universal constant, approximately 4.67—now known as the Feigenbaum number—which appears in chaotic systems where Chaos arrives via period doubling, systems like our dripping faucet. This amazing result tells us when to anticipate changes in dynamics. For instance, if our first transition happens when the mirror is 12 inches out, and the second transition occurs at 8 inches, we note the **difference** in these parameter values, 4 inches (Figure 8). Feigenbaum tells us that we ought to expect another transition $(d_1 - d_2)$ in another $4 / 4.67$ inches, or 0.85 inches from the point of the last transition. Now, in any system where we try to make predictions this way, we may face other complications. Our moving mirror, for example, may actually change several control parameters at once, such as brightness and focus. However, the mere existence Feigenbaum constant gives us hope for anticipating critical changes in complicated systems; in fact you should find that this prediction **works** for your measurements with your nightlight system!

This third home demo brings to light several key results that generalize to many chaotic systems. In particular, the demonstration illustrates:

1. the potential dynamics we can generate by imposing **feedback** on a system;
2. very specific behaviors to **expect** in chaotic systems;
3. suggestions of ways to **control** a system's dynamics;
4. ways to analyze and control a system with **no need for a formula** or model;
5. how the Feigenbaum constant helps **anticipate system transitions**.

Other Home Demonstrations

Many other systems you see every day exhibit chaotic dynamics. Watch the cream stir into your coffee. How does a stop sign wobble in a rough wind? Think about the position and speed of a car along a major city's beltway. What are the states of **all** the cars traveling the beltway?¹³ Watch the loops and spins of a tire swing in a park. If you're really adventurous, hook up your home video camera so it shows a live picture on your television set. Then, aim the camera at the television set and zoom in and out to generate some exciting feedback loops.

Consider how you might carefully describe those **systems**. What can you **observe and measure** in those systems; what are the important **parameters**? As the control parameters increase or decrease, what transitions in behavior should you expect?

I've summarized several **home** demonstrations in this chapter to introduce some intuition, as well as the vocabulary and tools of dynamical systems. I hope they spark your imagination about comparable systems that interests you. More important, they may represent your first **experience** with chaotic systems, so you can begin to **expect** and **anticipate** Chaos in **your** systems. The next chapter adds more detail to the vocabulary and results introduced here.

TWO

Definitions, Tools and Key Results

Of all the possible pathways of disorder,
nature favors just a few.¹⁴

- James Gleick

Introduction

The previous chapter described a few simple demonstrations so we could begin to develop some basic intuition for chaotic dynamics. I also used some of the associated Chaos vocabulary in those demos in order to introduce the definitions in the context of real systems. **Detailed** definitions require too much time to present in full. However, we need to spend time reviewing some vocabulary with care, since the **tools** to observe and explore complex dynamics are linked closely to the vocabulary we use to describe our observations. Rather than pore through excruciating details of precise definitions, this

chapter concentrates on the **consequences** of the definitions. My focus will be to answer questions such as, “What does it **mean** to be an **attractor**?”

We’ll narrow the discussion to the most important issues: What **is** Chaos? How can we test for it? What does it **mean** to me if I have Chaos in my system? By concluding with a summary of Chaos Theory’s key results, we’ll pave the way for later chapters that suggest ways to apply those results.

For this chapter, I’ll rely on two classic chaotic systems: the logistic map and Lorenz’ equations for fluid convection. These two examples reinforce some of the lessons we learned in the last chapter, and they make a nice bridge to the military systems we’ll examine in the next chapter. In particular, I’ll apply common Chaos tools to these two examples, so you can visualize the kind of new information Chaos Theory can provide about a system’s behavior.

The Logistic Map

What’s the system? In the early seventies, biologist Robert May researched the dynamics of animal populations. He developed a simple model that allowed for **growth** when a population of moths, for instance, was small; his model also **limited** population growth to account for cases of finite food supply.¹⁵ His formula is known as the **logistic equation** or the **logistic map**.

What are we observing and measuring? The logistic map approximates the value of next year's population, $x[n+1]$, based on a simple quadratic formula that only uses information about this year's population, $x[n]$:

$$x[n+1] = \lambda x[n] (1 - x[n]). \quad (2)$$

The parameter λ quantifies the population growth when $x[n]$ is small, and λ takes on some fixed value between 0 and 4. In any year n , the population $x[n]$ is measured as a **fraction**, between 0 and 1, of the largest community possible in a given physical system. For example, how many fish could you cram into the cavity filled in by a given lake? The population $x[n]$ expresses a **percentage** of that absolute maximum number of fish.

It's not too hard to illustrate the dynamics of the logistic map on your home computer. Even with a spreadsheet program, you can choose a value for λ and a starting value for $x[1]$, and calculate the formula for $x[2]$. Repeated applications of the formula indicate the changes in population for as many simulated "years" as you care to iterate. Some of the dynamics and transitions you should expect to see will be discussed in this chapter.

What's the significance? One helpful simplification of May's model was his approximation of continuously changing populations in terms of **discrete time intervals**. Imagine, for instance, a watch hand that jerks forward second by second instead of gliding continuously. Differential equations can describe processes that change smoothly

over time, but differential equations can be hard to compute. Simpler equations like the logistic map, **difference equations**, can be used for processes that jump from state to state. In many processes, such as budget cycles and military force reductions, changes from year to year are often more important than changes on a continuum. As Gleick says, “A year-by-year facsimile produces no more than a shadow of a system's intricacies, but in many real applications the shadow gives all the information a scientist needs.”¹⁶

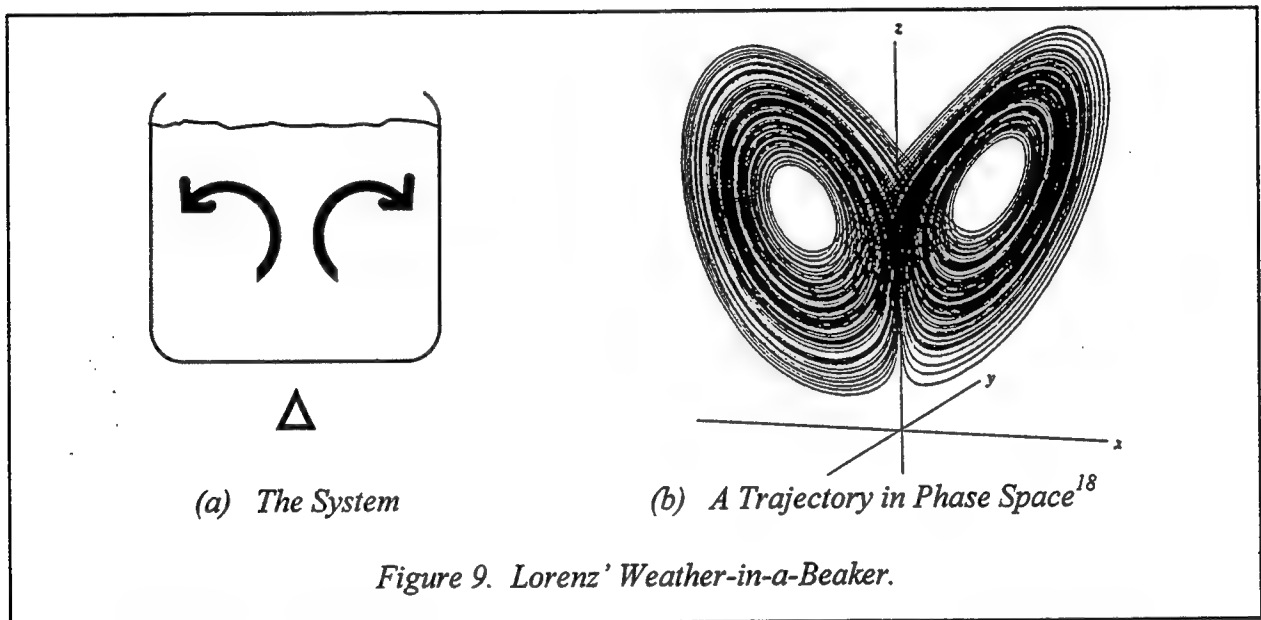
The additional beauty of the logistic map is its simplicity. The formula includes nothing worse than an x^2 term—how badly can this model behave? Very shortly, you’ll find that this simple difference equation produces **every** significant feature common to most chaotic systems.

The Lorenz Equations

What’s the system? Meteorologist Edward Lorenz wanted to develop a numerical model to improve weather predictions. Focusing on a more manageable laboratory system, the convection rolls generated in a glass of heated water, Lorenz modified a set of three fairly simple differential equations:¹⁷

$$\begin{aligned}x' &= -\sigma x + \sigma y \\y' &= Rx - y - xz \\z' &= -Bz + xy\end{aligned}\tag{3}$$

What are we observing and measuring? The phase variables, x , y and z combine measurements of the flow, as the heated water rises, cools, and tumbles over itself (Figure 9). The x variable, for instance, is proportional to the intensity of the convection current; y is proportional to the temperature difference between the rising and falling currents. The numbers σ , R and B are the system's physical parameters, which Lorenz set at $\sigma = 10$, $R = 28$, and $B = 8 / 3$. As the phase variables change in time, they trace out fascinating patterns, like those illustrated in Figure 9b.



What's the significance? The Lorenz equations crudely model only one simple feature of fluid motion: temperature-induced convection rolls. However, even in this simple system, Lorenz observed extreme sensitivity to initial conditions, as well as other symptoms of Chaos we'll see momentarily. He clearly proved that our **inability to predict** long-term weather dynamics was **not** due to our lack of data. Rather, the

unpredictability of fluid behavior is an **immediate consequence of the nonlinear rules that govern its dynamics.**

Definitions

Now that we have two new systems to work with, along with the “experience” of our home demonstrations, let’s highlight the vocabulary we’ll need to discuss more complicated systems.

Dynamical System. Recall how we defined a **system** as a collection of parts along with some recipe for how those parts move and change. We use the modifier **dynamical** to underscore our interest in the **character of the motions and changes**. In the case of the logistic map, for example, the **system** is simply a **population** measured at regular time intervals; the logistic **equation specifies how this system changes** in time.

Linear and Nonlinear. The adjective **linear** carries familiar geometrical connotations, contrasting the **linear** edge of a troop deployment, for example, with the **curved** edge of a beach. In mathematics, the concept of linearity takes on broader meaning to describe general **processes**. We need to understand linearity because **isolated linear systems can not be chaotic**. Moreover, many published explanations of linearity make serious errors that may prevent you from grasping its significance.

Some authors condense the definition of linearity by explaining that, in a linear system, the output is proportional to the input. This approach may be helpful when we model the lethality of certain aircraft, saying that three sorties will produce three times the destruction of a single sortie. However, I want you to get at least one more level of deeper insight into linearity. I'll build that insight from our first home demonstration, the pendulum.

Even though a pendulum swings in a curve, and we describe its motion with sine and cosine functions, an ideal pendulum is a **linear** system! It's linear because the **equation that defines its motion has only linear operations**: addition, and multiplication by constants. Common **nonlinear** operations, of course, include exponents, trigonometric functions and logarithms. The important consequence for us is that the solutions to most linear systems are **completely known**. This may not seem earth-shattering for a single pendulum, but many oscillating systems—such as vibrating aircraft wings, mooring buoys, and concrete structures subjected to shock waves—behave just like a collection of coupled pendula. Therefore, as long as they aren't regularly “kicked” by external forces, those real systems are just enormous **linear systems** whose range of possible motions is **COMPLETELY KNOWN!**

Without delving into the subtleties of more analytical definitions, here are some important consequences of the property of **linearity**:

- The solutions to linear systems are **known** (exponential growth, decay, or regular oscillations), so **linear systems can't be chaotic**.

- “Kicking” or forcing an otherwise linear system **can** suffice to drive it into Chaos.
- If you observe Chaos in a system, there **must** be some underlying nonlinear process.

This discussion of linearity should serve as a wake up call. Basically, if you have a system more complicated than a pendulum, or if you see an equation with nonlinear terms, you should be alert for possible transitions from stable behavior to Chaos. This is certainly a simplification, since many systems include control mechanisms that stabilize their dynamics, such as feedback loops in human muscles or in aircraft control surfaces. However, the minimum ingredients that make Chaos possible are usually present in systems like these. In the absence of any reliable control, unpredictable dynamics are not difficult to generate.

Phase Space and Trajectories. A system consists of components and their rules of motion. To analyze a system we must also decide exactly what **properties** of those components we will measure and record. All the time-dependent properties necessary to determine the system dynamics are known as the system’s **phase variables**. The collection of **all possible combinations** of values those variables can attain is then the **phase space** for our system.

Phase space is the canvas where we paint a system’s dynamics. The Lorenz equations, for example, define the time-dependent changes of fluid flow in a heated beaker of water. If we start at some initial state and let the system evolve in time, we can

keep track of how the three system variables change. We can then plot that information with a three-dimensional curve (Figure 9b). Notice that the curve does **not** directly illustrate the physical motion of the water. Rather, the curve indicates changes in all three phase variables; at least one of these—the temperature gradient, y —quantifies changes we can't see. The plot's **entire** three-dimensional space constitutes the **phase space** for the Lorenz equations; we call the single curve that leaves a particular initial state the **trajectory** (or **trajectory in phase space**) for that initial condition.

Parameter. A **parameter** is a quantity that appears as a **constant** in the system's equations of motion. The logistic map has only one parameter, λ , which expresses the rate of growth for small populations. A pendulum's parameters include its mass, the length of its string. Sometimes, a parameter expresses a **physical constant** in the system, such as the gravitational constant for the pendulum. Most important, a system parameter often represents a **control knob**, an opportunity for us to **control the amount of energy in a system**.

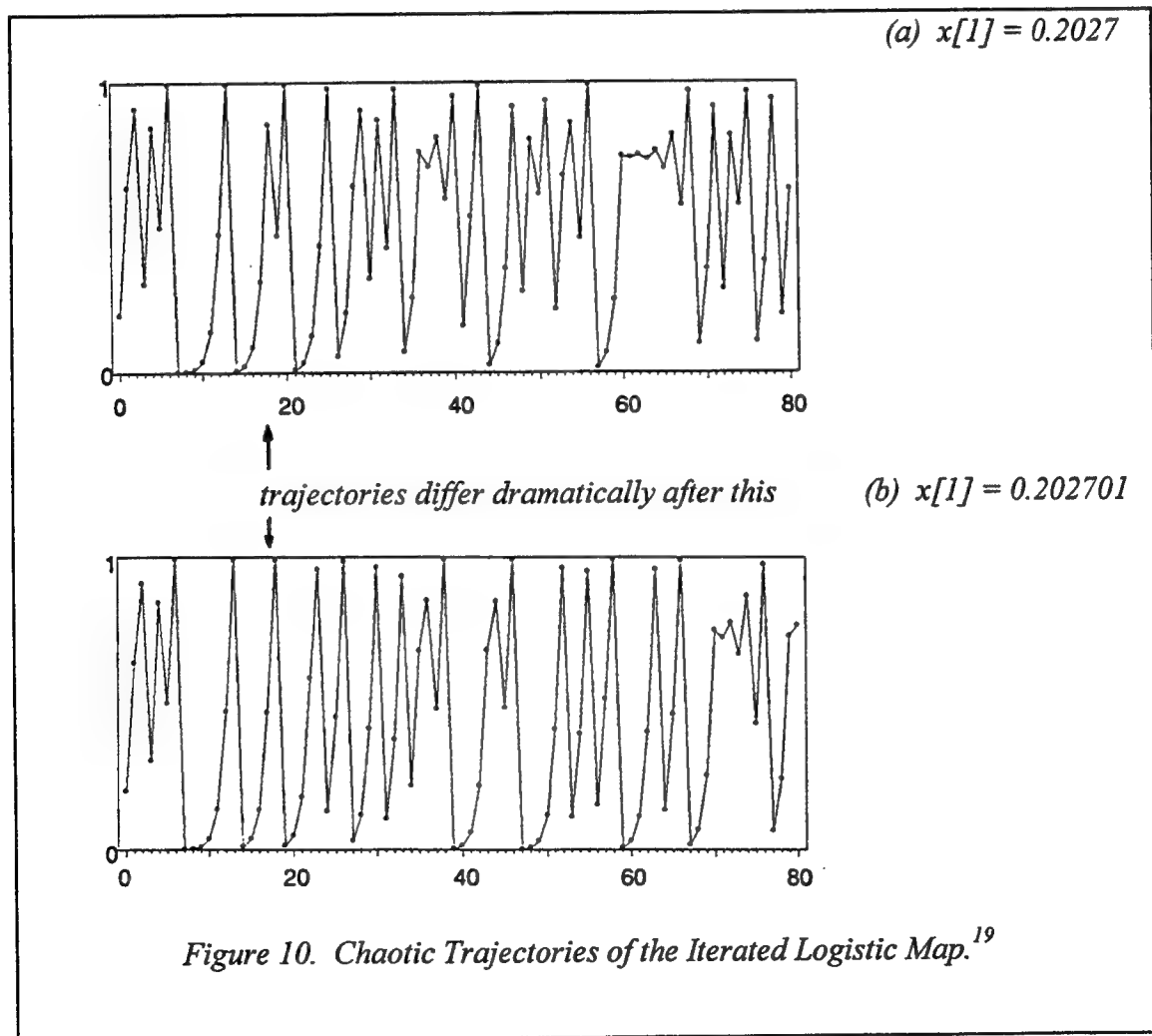
For instance, we saw earlier how changes in flow rate, the key parameter for the dripping faucet, drove transitions in system output. In the following section on Chaos Tools, we'll see how the logistic map undergoes transitions as we increase λ from 0 to 4. It's important to note that, even in relatively simple systems like the faucet, there are many influential parameters that are not easily controlled: spout diameter, mineral content of the water, local humidity, spout viscosity, etc. One crucial skill for any decision maker

is the ability to **identify all the parameters accessible** to external control, and to **isolate** those parameters that have the greatest influence on a system.

Sensitivity to Initial Conditions (SIC). Any system “released” from its initial state will follow its laws of motion and trace out some trajectory in phase space, just as we saw with the logistic map above. However, if a system is SIC, we also know that **any two** initial states that deviate by the slightest amount **must** follow trajectories that diverge from each other exponentially. Look at Figure 10. The lower time series started from an initial population only slightly greater than the upper case; after about 16 iterations, the two trajectories bear no resemblance to each other. This is an illustration of SIC.

We can **measure how fast neighboring trajectories tend to diverge**. At any given point in phase space, a ***Lyapunov (lee-OP-uh-noff) exponent*** quantifies this **rate of divergence**. This exponent has properties that come from its role as the constant **k** in the exponential function $e^{k \cdot t}$: if **k** is negative, then small disturbances tend to get smaller, indicating no SIC; if **k** is positive, small perturbations increase exponentially. With these measurements, we can get a handle on how “touchy” a system is, how **vulnerable** the system may be to external disturbances, and **how unpredictable** the consequences of our actions may be. We can often calculate an **average Lyapunov exponent** for an entire region of phase space. This value gives us a means to compare two systems, or two scenarios, and decide which one tends to be more or less predictable. Information like this could prove valuable for prioritizing the courses of action available to a commander.

Many systems are SIC, including some non-chaotic systems. For example, take the simplest case of exponential growth, where a population at any time t is given by a recipe such as: e^{3t} . This system is SIC, but certainly not chaotic. What does this mean for us? If your system is SIC, you're **not** guaranteed to find Chaos. However if your system is **not** SIC it **can not** exhibit Chaos. Thus, we've identified SIC as a **necessary but not sufficient** condition for Chaos to occur.



Attractors. Despite the fact that chaotic systems are SIC, and neighboring trajectories “repel” each other, those trajectories still confine themselves to some limited region of phase space. This **bounded** region will have maximum and minimum parameter values beyond which the trajectories will not wander, unless perturbed. In the logistic equation, the population remains bounded between the extreme values of 0 and 1, though it seems to take on every possible value in between, when it behaves chaotically.

In the Lorenz equations, the trajectories also stay within finite bounds, but the trajectories do not cover **all** the possible values within those limits. Instead, a single trajectory tends to trace out a complicated, woven surface that folds over itself in a bounded region of phase space (Figure 9). The collection of points on that surface is an **attractor** for those dynamics; the classic Lorenz attractor is a particularly striking example.

Left to itself, a single trajectory will always return to revisit **every** portion of an **attractor**, unless the trajectory is perturbed. All chaotic, or **strange**, attractors have this **mixing property**, where trajectories repeatedly pass near every point on the attractor. Just envision where a single drop of cream goes as you stir it into your coffee.²⁰ You could also imagine the path of a single speck of flour as you knead it into a ball of dough. If you continue mixing long enough, you’d expect the small particle to traverse every neighborhood of its space. Actually, one of the best ways to sketch a rough image of an attractor is simply to plot a single trajectory in phase space for a **very** long time.

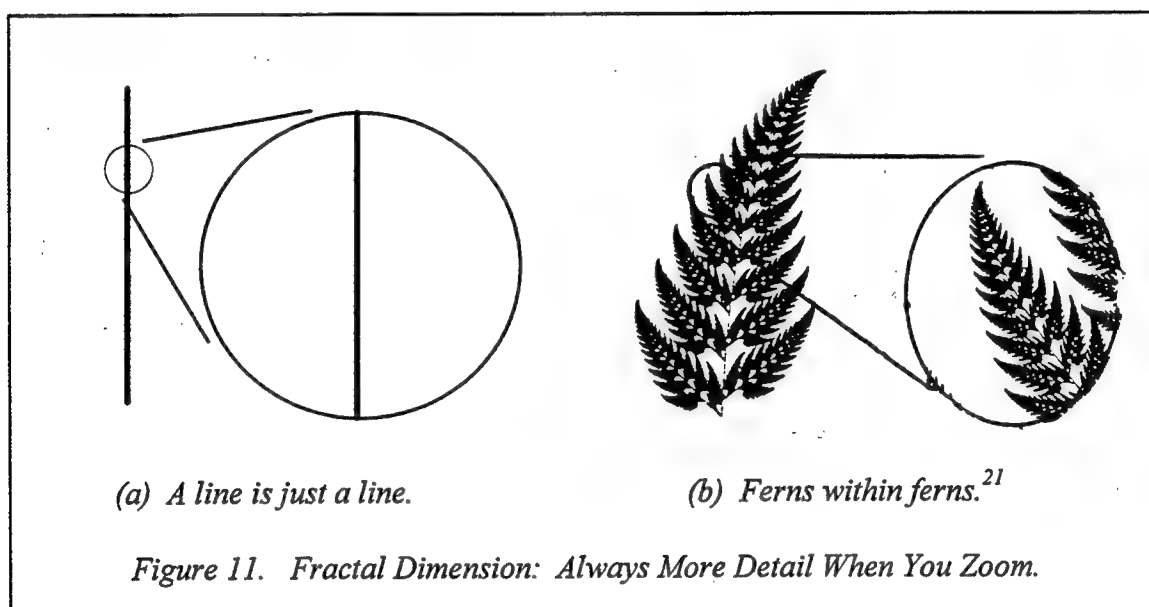
Transient states are all the initial conditions **off the attractor** that are **never** revisited by a trajectory. If we gather together all the transient states that eventually evolve toward a single attractor, we define the **basin of attraction** for that attractor. Thus, the **basin** represents all the possible initial states that ultimately exhibit the same **limit dynamics** on the attractor. In the Lorenz system, for instance, we might **start** the system with a complicated temperature distribution by dropping an ice cube into hot water. However, that transient extreme will die out and, after a while, the system **must** settle down onto the collection of temperature variations that stay on the attractor. Because of SIC, we can't predict the precise state of the Lorenz system at any given time. However, because the attractor draws dynamics toward itself, we **do** know what the **trends in the dynamics** have to be!

When we examine those trends closely, we find that a single trajectory visits different regions of the attractor **more often** than others. That is, if we color each point on the attractor based on how often the trajectory passes nearby, we'll paint a richly detailed **distribution** of behavior on the attractor. To picture this, visualize the distribution of cars on the interstate beltway around a big city. At any point, on a given day, we could note the number of vehicles per mile, and begin to identify patterns of higher traffic density for certain times of day. You could continue and consider the distribution of cars on whatever scales interest you: all interstates; all streets; just side streets. Even though you can't predict the number of cars present on any particular street, these distributions and patterns give you crucial information on how the overall system **tends** to behave.

The properties of attractors are key signposts at the junction where Chaos Theory matures past a mere metaphor and offers opportunities for practical applications. Attractors provide much more information than standard statistical observations. This is because an attractor shows not only distributions of system states, but also indicates “directional” information, that is, how the system tends to proceed away from its current state. As a result, **when we construct an attractor we reconstruct an image of the system’s global dynamics—without appealing to any model!** In subsequent chapters, we’ll see how this reconstruction allows us to **predict short-term trajectories and long-term trends**, to perform **pattern recognition**, and to carry out **sensitivity analysis** to help us make **strategic decisions**.

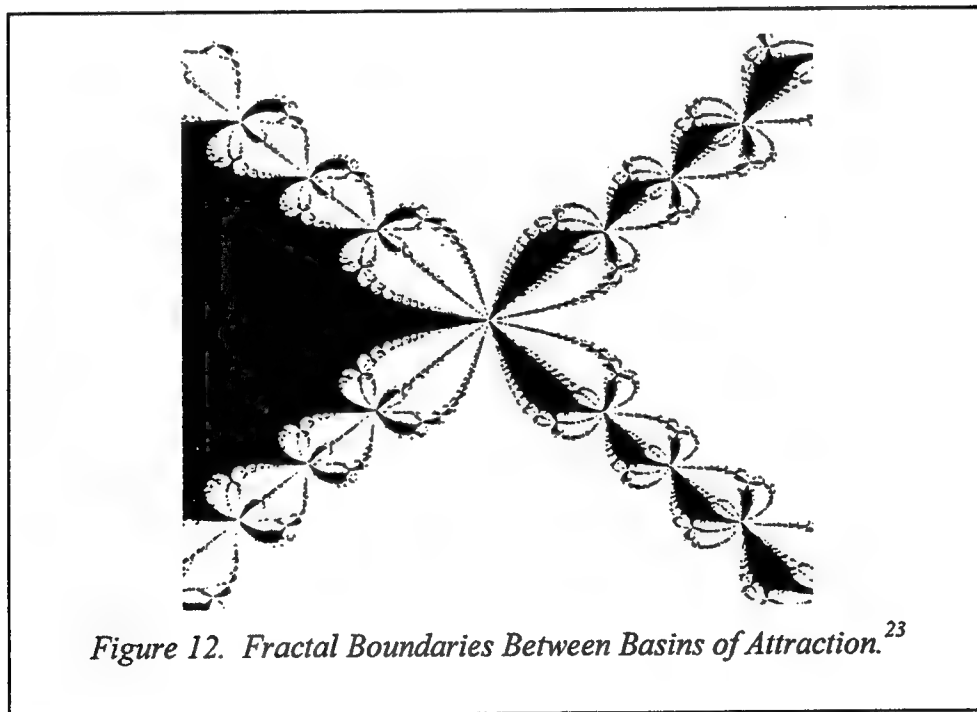
Fractal. Though there are standard definitions of several types of **fractals**, the important consequence for us is that **fractals** describe the complexity, or the amount of detail, present in objects or data sets. If we think of a well-defined line, like the y-axis on a graph, we call it **one-dimensional** because one piece of information, the y-coordinate, suffices to pinpoint every position on the line. To get an idea of what **dimension** means in a fractal sense, imagine **zooming in** on a line, with a microscope. However intently you zoom in, the most detail you can expect to see is a razor thin line cutting across your field of view (Figure 11a). If you focus your microscope, instead, on a **two-dimensional object** like a square, sooner or later your narrow field of view will fill with an opaque image. You need **two coordinates** to pinpoint any place on that image.

On the other hand, a **fractal image** has a non-integer dimension. An image with dimension 1.7, for instance, has more detail than a line, but too many holes to be worthy of the title **two-dimensional**. Fractal images contain **infinite detail** when we zoom in (Figure 11b). The good news is that the extraordinary detail present in fractal images can be generated by **very simple recipes**.



The study of fractal geometry becomes important to military applications of Chaos in three main areas: image compression, dimension calculations, and basin boundaries. In **image compression**, the infinite detail generated by simple sets of instructions allows us to compress images by transmitting the **instructions** rather than pixel-by-pixel images. The second application, **dimension calculation**, is possible with time series as well as with geometric figures; when we calculate the **dimension** of a sequence of data points, we get an estimate of the **minimum number of variables** needed to model the system from which we measured the data! Finally, many systems

that have two or more attractors also have two or more basins of attraction. Very often, the **boundaries between basins** are not smooth lines. Instead, the basins overlap in fractal regions where one initial condition may lead to steady state behavior, but **any** nearby initial condition could lead to completely different behavior. Consider the illustration in Figure 12, the basins of attraction for a numerical model. All the initial conditions shaded white lead to one kind of behavior; all the dark points lead to entirely different behavior. A commander making decisions in such an environment will have to be alert—small parameter changes, in certain regions, produce dramatic differences in outcomes.²²

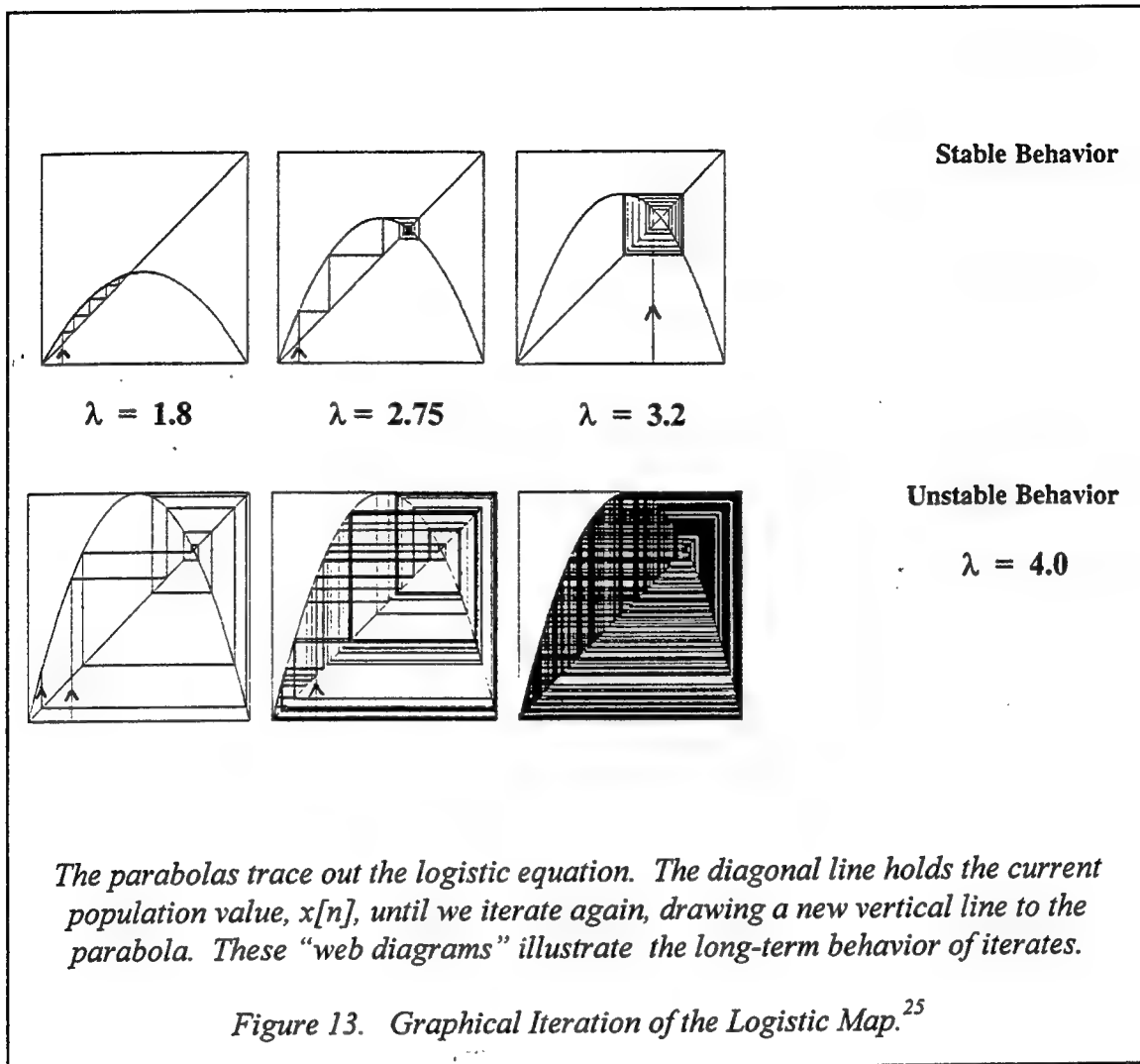


Bifurcation. Bifurcation theory represents an entire subdiscipline in the study of dynamical systems. I mention bifurcations here for two reasons. First, so you will

recognize the word in other references. In the context of the demos we've seen so far, a **bifurcation** is simply a **transition in dynamics**. Our faucet, for example, drips slowly when the flow rate is low. At some higher flow rate, the drops come out with period-2; we say the system has undergone a bifurcation from one kind of periodicity to another. Notice that a bifurcation is a qualitative transition in system dynamics due to a change in a **control parameter**.

The second reason I surface this new vocabulary word is to highlight the **universality** of bifurcation types. That is, when you modify one of your system parameters, you may see subtle bifurcations, or catastrophic bifurcations, but a few **classes of bifurcations** are **common** to many dynamical systems.²⁴ Recall the discussion of transitions in our home demo with the night-light. The transitions came at smaller and smaller intervals, roughly according to patterns predicted by the Feigenbaum constant. Well, Mitchell Feigenbaum first discovered this constant through his study of the logistic map, where transitions occur in the same pattern as in the night-light. Overall, the most important consequence is that many **transitions** in behavior **are universal** in apparently unrelated physical systems.

Dense, Unstable, Periodic Orbits. Let's look at one last feature of the logistic map that ultimately makes it possible for us to **control chaotic systems**. I'll talk much more about Chaos control in the next chapter. For now, be patient as we take a few steps through the dynamics of the logistic map in order to glimpse the complicated activity on an attractor, as illustrated in Figure 13.



Suppose you set the parameter to a small value, say $\lambda = 1.8$. You can start the system with $x[1]$ anywhere between 0 and 1, and successive iterations of the logistic equation will always drive the value of $x[n]$ toward 0.44, a stable fixed point. If we increase λ to 2.75, the system still has a stable fixed point, but that point is now around 2/3. We raised the control parameter, but we observed no qualitative changes in behavior. However, if we raise λ slightly above 3, the system does **not** settle into a

fixed point, but falls into a cycle of **period-2**. Thus, at $\lambda = 3$ we see a **bifurcation** from stable to periodic behavior.

Transitions come hand-in-hand with **changes in stability**. Any system might have both stable and unstable behaviors. The equations governing a pencil standing on its point have a good mathematical solution, with the center of gravity directly above the point—but **you** cannot stand a pencil on its point because that state is **unstable**. That is, the slightest perturbation draws the system away from that state. On the other hand, a marble lying at the bottom of a bowl stays there, because if the marble is perturbed slightly in any direction, it just rolls back.²⁶

The important feature for us hides in the chaotic trajectory smeared out in Figure 13, when $\lambda = 4$. Inside that smear—the attractor for this chaotic system—many **periodic** cycles still exist, on paper, that is. The fixed point, for instance, still lives at the place on the graph where the parabola intersects the diagonal. However, that point is **unstable**, so a **trajectory can never approach it**. Similarly, we can **calculate** trajectories of period-2, period-3, actually every possible period. In fact, there are **infinitely many** unstable, periodic trajectories woven through the attractor, woven thickly in a way mathematicians call **dense**. That means every area surrounding **every point** on the attractor is **crowded** with these “repelling” unstable, periodic trajectories.

So, on one hand, it’s not useful to locate any of these periodic behaviors, because **all** these trajectories are **unstable**. On the other hand, recent experiments have demonstrated ways to **force the system to follow one of these periodic behaviors**. This

is the power of **Chaos control**; as we'll see later, the density of these trajectories is the property that makes this control possible.

So How Do We Define Chaos?

A chaotic system **MUST** be:

- bounded;
- nonlinear;
- non-periodic;
- sensitive to small disturbances;
- mixing.

A chaotic system **usually** has the following observable features:

- transient and limit dynamics;
- parameters (control knobs);
- definite transitions to and from chaotic behavior;
- attractors (often with fractal dimensions).

What's the significance of these properties? Measurements of transient and limit dynamics in a system provide **new information** not available to us before the advent of

Chaos Theory. Our comprehension of the role of parameters in system dynamics offers opportunities for **new courses of action**, to be described in subsequent chapters. Finally, the common properties of system transitions and attractors suggest **new expectations** of system behavior, as well as **new strategies** for coping with those expectations. For other, more detailed characteristics of chaotic data—such as exponentially decaying correlation, and broad power spectra—you can refer to any one of the texts described in Chapter 5: *Suggestions for Further Reading*.

This is, perhaps, not so much a definition, as it is a list of **necessary ingredients** for Chaos in a system. That means, without any one of these properties, a system can not be chaotic; I believe my list is also **sufficient**, so if a system has **all** these properties, it can be **driven** into Chaos.

Random. You may look at the above definition of Chaos and wonder if the processes we call **random** have those same properties. For those interested in more detail, a discussion of one definition of **random** appears in the Appendix. However, I'll pause here to focus on one **difference** between **random** and **chaotic** dynamics. Please be aware that I'm ignoring some large issues debated by Chaos analysts. Some argue, for instance, that the kind of dynamics we now call "random"—like a roulette wheel—simply come from **chaotic** systems, with **no** random variables, where we just don't know the model. In other cases, "noise," or random imperfections in our measurements—like radio static—may come from Chaos that happens on a scale we haven't yet detected. For our purposes, the **primary feature distinguishing chaotic from random** behavior is the

presence of an attractor that outlines the dynamics towards which a system will evolve.²⁷ Existence of such an attractor gives us **hope that these dynamics are repeatable**.

In the water drop experiment, for example, if results were random, the experiment would not be repeatable. However, if you and I both run this test, I can list my experimental parameters for you—such as nozzle diameter and flow rate—and the key features of this system's dynamics will be replicated precisely by our two separate systems. Slow flow is **always** periodic. The system undergoes **period doubling** (period-2, then period-4, . . .) on the way to Chaos, as we increase the flow rate. Most important, for high flow rates, **your** chaotic return map for time differences between drops will produce a smear of points **nearly identical** to mine! If the system were exhibiting random behavior, these **global features** would **not** be reproducible.

The Chaos Con

Before we leave this review of basic Chaos vocabulary, we need to examine the common mistakes and misrepresentations that appear in many papers on the subject. The sum of these errors constitutes **The Chaos Con**, the unfortunate collection of misleading publications that tend to crop up when new researchers investigate new topics. Everybody's taken up the best **titles** for articles, and **published** them, without always presenting the most thorough results. The Con may come from well-intentioned authors, new to the subject, who miss some key concepts because they're constrained by time.

Other Cons may come in contract proposals from cash-starved analysis groups who might try to dazzle you with the sheer volume of their Chaos vocabulary. It's very important to avoid The Con, both intentional and innocent, but most of all, don't con **yourself** by making any of the following common errors!

"Chaos is too difficult for you." Don't let anyone fool you: if you finished college, you can follow the basics of Chaos. Be suspicious of anyone who tries to tell you that the **general** concepts are beyond your grasp. Some authors will disguise this false claim with subtle references to the "mysteries of Chaos" or "mathematical alchemy" or other vocabulary designed to intimidate their readers. Don't believe it, and **don't pay these folks** to teach you Chaos. You **can** learn it—just remember to take your time.

"Linear is. . ." Remember that some writers will oversimplify the definition of **linearity** by waving their pen quickly at some phrase like "output is proportional to input." That comment is only true if a system's output and input are **very** carefully defined. Never forget that pendula, swings, and springs are **all linear systems!** Make sure your author's definition for linearity admits these three important physical systems.

Bifurcation. What exactly bifurcates? Trajectories **don't** bifurcate, as I've seen some authors claim. A single trajectory can only do one thing. We may have a limited capacity to **predict** that behavior, but—like a light bulb can only be on or off at any fixed time—a single system can only evolve through one state at a time. Remember that a

bifurcation is a qualitative change in system behavior that we observe as we change **parameter settings**.

*“Complicated systems **MUST** be chaotic.”* Just because a system is complicated or has many components, that system does not **necessarily** allow Chaos. For instance, many large systems behave like coupled masses and springs, whose linear equations of motion are completely predictable. Similarly, other large systems include reliable control mechanisms that damp out perturbations and do not permit sensitive responses to disturbances. Such systems do **not** exhibit Chaos.

*“We **NEED** many variables for Chaos.”* Many of the same authors who claim big systems must be chaotic, also propagate the fallacy that simple systems can not exhibit Chaos. Nothing could be further from the truth. In fact, the power of Chaos Theory is that the simplest interactions can generate dynamics of profound complexity. Case in point: the logistic map produces every symptom of Chaos described in this paper.

“Butterflies cause hurricanes.” When Edward Lorenz presented his findings of SIC in weather systems, he described **The Butterfly Effect**, the idea that the flapping wings of a butterfly, in one city, will eventually influence the weather patterns in other cities. This phenomenon is a necessary consequence of the sensitivity of fluid systems to small disturbances. However, The Butterfly Effect often gets lost in the translation. Be wary of authors who suggest that a butterfly’s flap in Florida will become **amplified** somehow

until it spawns a **hurricane** in California! Believe it or not, several often-cited reports make this ridiculous claim. Make no mistake, if a weather system has enough energy to produce a hurricane, then the storm's **path** will be influenced by butterfly aerodynamics across the globe. However, the system **does not amplify** small fluid dynamics; rather, it amplifies our inability to **predict** the future of an individual trajectory in phase space.

“Chaos” versus “chaos”. One of the first signals of a weak article is when the author inconsistently mixes comments on mathematical Chaos and social chaos. If we can't distinguish between the two, we can't get past the metaphors of Chaos to practical applications. As we'll discuss below, the existence of Chaos brings guarantees and expectations of specific phenomena: attractors; complex behavior from simple interactions; bounded, mixing dynamics; universal transitions from stable to erratic behavior.

The worst consequence of The Chaos Con is that the well-intentioned reader may not discern the important **results** of Chaos Theory. These results highlight the common characteristics of chaotic dynamics that provide a useful template for the kinds of dynamics and applications we should **expect** in a chaotic system. A discussion of the results I consider most important follows here; my explanation of their applications constitutes the remaining portion of this essay.

TOOLS of Chaos Analysts.

One of the most important outcomes of the study of Chaos Theory is the extraordinary array of tools that researchers have developed in order to observe the behavior of nonlinear systems. I can not emphasize enough that these tools are designed not only for **simulated systems**. We can calculate the same information from time series measurements, **when there is no model available, and often when we can only measure one variable in a multi-variable system!** Moreover, decision makers need the skills to **differentiate** random behavior and Chaos, because the tools that allow us to understand, predict and control chaotic dynamics have no counterpart in random systems.

For the military decision maker who needs these tools, two issues stand out:

What are the preferred tests for deciding if a system is chaotic?

How can we tell the difference between random and Chaos?

The analytical tools used by Chaos analysts answer these questions, among many others. This brief summary of the most basic tools begins with an important reminder. We always need to begin our analysis by answering two additional questions: what is the system, and what are we measuring? For example, recall the dripping faucet system, where we observe the dynamics, not by measuring the drops themselves, but by measuring **time intervals** between events. Only after we answer those two questions should we move on to consider some of the qualitative features of the system dynamics:

- What are my **parameters**? Can I control their magnitude?
- Does the system perform many **repetitions** of its events?
- Are there inherent **nonlinearities** or sources of feedback?
- Does the phase space appear to be **bounded**? Can we prove it?
- Do we observe **mixing** of the phase variables?

When we have a good grasp of the general features of our system, we can begin to make some measurements of what we observe. We should note, however, that our aim is not merely to passively record data emitted from an isolated system. Very often, our interest lies in **controlling** that system. In an article on his analysis of brain activity, Paul Rapp summarizes:

*Quantitative measures [of dynamical systems] assay different aspects of behavior, and they have different strengths and weaknesses. A common element of all of them, however, is an attempt to use mathematics to **reconstruct the system generating the observed signal**. This contrasts with the classical procedures of signal analysis that focus exclusively on the signal itself.²⁸*

Therefore, keep in mind that the tools presented here are not used for observation only. They provide the means to **re-create a system's rules of motion**, to **predict that motion**, over short time scales, and to **control that motion**.

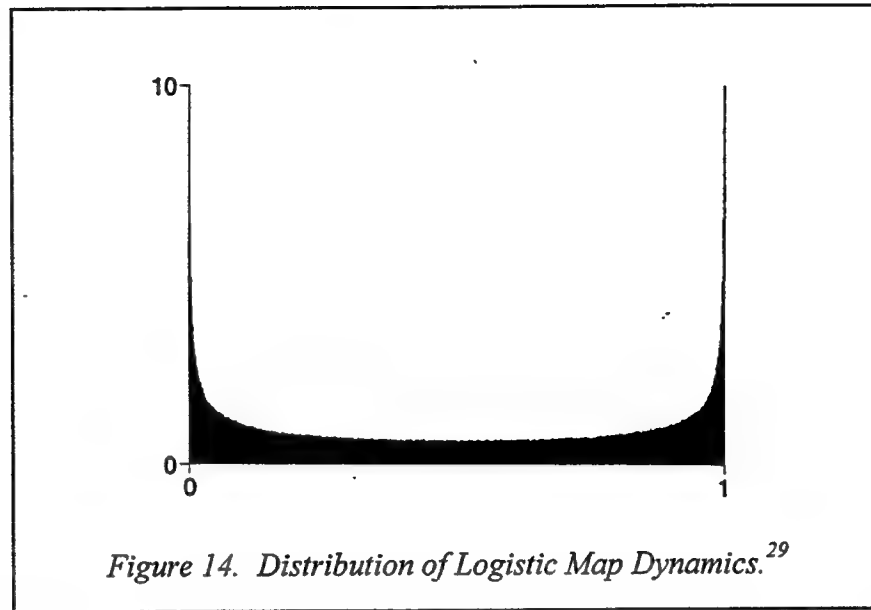
Depicting Data. We've already encountered most of the basic tools used for observing dynamical systems. The two simplest tools—**time series plots** and **phase**

diagrams—display raw data so we get a qualitative picture of the data's bounds and trends. A **time series plot** graphs a sequential string of values for **one selected phase variable**, as in the plot of population variation for the logistic map in Figure 10. Sequential graphs give us some intuition for long-term trends in the data and for the system's general tendency to behave periodically or erratically.

Phase diagrams trace the dynamics of several phase variables at the same time, as the Lorenz attractor does in Figure 9. The first piece of information apparent from a good phase diagram is the nature of the system's attractor. The attractor precisely characterizes long-term trends in system behavior—how long the system spends in any particular state. This information translates directly into probabilities.

Attractors and Probabilities. As a demonstration of translating attractor dynamics into probabilities, consider the chaotic trajectories of the logistic map shown in Figure 13. The awful smear of trajectories makes it obvious that the population $x[n]$ takes on most of the values between 0 and 1, but is the smear of values **evenly distributed** across that range? One way to find out is to build a quick histogram: divide the interval from 0 to 1 evenly into a few hundred subintervals; keep a count of every time the evolving population $x[n]$ visits each subinterval. Figure 14 shows the results of such a calculation; we see from the figure that the trajectory of the logistic equation spends a great deal more time closer to 0 and 1 than it does near other values. To illustrate, if this equation modeled the number of troops assigned to a certain outpost, a distribution like this would

tell a commander that the site tends to be fully staffed or nearly vacant, with little probability of other incremental options.



Probability information like this has several immediate uses. First, of course, are the probability estimates that commanders require to prioritize diverse courses of action. Second, analysts can use this information to compare models with real systems, to gauge how well the distribution of a simulated system relates to real data. Third, since many simple chaotic systems use **non-random** formulas to generate **distributions** of behavior, the resulting distributions can be used in various simulations, to replace black-box random number generators. I'll discuss these applications in greater detail in Chapter Four.

Attractors and Sensitivity. As a single trajectory weaves its way through its attractor, we can also calculate local Lyapunov exponents at individual points on the attractor, as well

as an average Lyapunov exponent for the entire system. Remember, this exponent measures **how sensitive** trajectories are to small disturbances. Therefore, details about these exponents can guide decision makers to particular states where a system is more or less vulnerable to perturbation. The same exponents can also be calculated for various ranges of **parameter settings**, so commanders can discern which variables under their control may produce more predictable (or unpredictable) near-term outcomes.

Embedding. However directly we might calculate system features like attractors and Lyapunov exponents, how is it we can apply these tools to a real system where we have no descriptive model? Suppose we have a complicated system—like the dripping faucet—that gives us a time series with only one variable. What can we do?

The answer comes from a powerful technique known as **embedding**. Very simply, we can start with a sequence of numbers in a time series, and instead of isolating them as individual pieces of data, we can group them in pairs. The resulting list of pairs is a list of **vectors** that we can plot on a **two-dimensional** graph. We can also start over and package the data in groups of three, creating a list of vectors we can plot in three-dimensions, and so on. This process **embeds** a time series in higher dimensions and allows us to calculate all the features of the underlying dynamics from a **single time series!**

The suggested reading list in Chapter Five offers several sources that discuss this technique in detail. For now, though, be aware of the power of embedding as a measuring instrument:

- By embedding a time series we can calculate the **fractal dimension** of a data set. Since random data have theoretically infinite dimension, and many chaotic systems have smaller dimensions, this is one of the first tools we use to distinguish noise from Chaos. Even more important, the dimension of a time series measures the amount of detail in the underlying dynamics and actually **estimates the number of independent variables driving the system**. So, when Tagarev measures a fractal dimension of 2.9 for a time series of aircraft sorties (Figure 2), he presents strong evidence that the underlying system is **not** random, but it may be driven by as few as **three key independent variables**.³⁰
- Recent studies of embedded time series have also uncovered ways to use the embedding as a vast, generalized grid through which we can interpolate to approximate a system's dynamics. In this way, researchers have made tremendous strides in **predicting the short-term behavior of chaotic systems!** I'll discuss more details of these results in Chapter Four.

And Much, Much More. . . . The above list of tools represents only a small sample of the standard analytical tools currently in use. Consult the references I highlight in Chapter Five to find complete discussions of these and other tools, such as: return maps, Poincaré sections, correlations, Fast Fourier Transforms, and entropy calculations. This

tremendous toolbox is the primary source of the **new information** that Chaos Theory brings to decision makers.

RESULTS of Chaos Theory

Let's pause to gather together the theoretical results scattered through these first two chapters. First, I'll summarize the common features of chaotic systems. Then, I'll review what it **means** for us to have Chaos in our systems.

Here is a brief snapshot of the common characteristics of Chaos, a sample of what to expect in a chaotic system. I've highlighted most of these characteristics in the examples we've seen up to this point.

- **We don't need much** in a system in order **for Chaos to be possible**. In most physical systems, whose smooth changes in time can be described by differential equations, all we need are three or more independent variables and some nonlinear interaction. In difference equations, like the logistic map, where change occurs at discrete time intervals, all we need is a nonlinear interaction.
- Most systems have **accessible parameters**, system inputs we can control to adjust the amount of energy in the system. We should expect systems to have qualitatively different behaviors over different parameter ranges.
- Surprisingly **common transitions**, from stable equilibria to periodicity and Chaos, occur in completely unrelated systems.
- Influential dynamics occur on many different **scales**. For instance, the cloud cover that concerns us during a combat operation is affected by the hyperactivity of butterflies across the globe. To understand the larger scale dynamics, we may or may not need to consider the smaller scales.

- **Attractors draw trajectories** towards themselves. So, if attractor exists (in an isolated system), and the state of your system is in that attractor's basin, the system can't avoid proceeding toward the attractor. Dynamics on the attractor represent **global trends** of the underlying system, and they set global bounds on system behavior. The density of trajectories on the attractor also reveals the relative **distribution of behavior**.
- Because of the trajectory mixing that takes place on attractors, the attractors are immersed in dense weavings of unstable periodic trajectories. The presence of these potential periodic behaviors makes **Chaos control possible**.

The universal nature of the above properties helps us answer a somewhat bigger question:

What does it mean to me to have Chaos in my system?

One consequence of understanding the results of Chaos Theory is that, if we are confident that a system can behave chaotically, then we **know** that it **must** have all the properties of Chaos. Some of these properties are hard to prove, but we “get them for free” if we know the system is chaotic. In particular, if a system is known to be chaotic, then we know, for example, that any models of that system **must** include nonlinear terms. We also know we have avenues available to **control** the system. That is, any attractor for that system is densely woven with unstable periodic trajectories toward which we can drive the system (see the discussion of Chaos Control coming in Chapter Four).

In a 1989 Los Alamos report, David Campbell and Gottfried Mayer-Kress summarize **their** “lessons of nonlinearity”:

1. **Expect** that nonlinear systems will exhibit **bifurcations**, so that small changes in parameters can lead to qualitative transitions to new types of solutions.
2. Apparently random behavior in some nonlinear systems can in fact be described by **deterministic [non-random] chaos**.
3. Typical nonlinear systems have **multiple basins of attraction**, and the **boundaries** between different basins can have incredibly complicated **fractal** forms.
4. Our heightened awareness of the **limits to what we can know** may lead to more care and restraint in confronting complex social issues.
5. The **universality** of certain nonlinear phenomena implies that we may hope to understand many disparate systems in terms of a few simple paradigms and models.
6. The fact that Chaos follows from well-defined dynamics with no random influences means that in principle **one can predict short-term behavior**.
7. The dense paths of trajectories on an attractor make **Chaos control** possible.³¹

To the above list, I’d add that a basic understanding of Chaos brings not only limits to what we can know, but, more important, **new information about the dynamics that are possible**. In the next chapter I’ll outline some common **military systems** where you can expect to see Chaos. Then, in Chapter Four, we’ll be ready to learn how to apply all these results.

PART II

Who NEEDS Chaos Theory?

Applications

**Big whorls have little whorls
Which feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity.**

- Lewis F. Richardson³²

**Thank heaven
For little whorls**

- not quite Maurice Chevalier

THREE

Expect to See Chaos

Specific Military Systems and Technology

Chaos Theory does not address **every** military system. However, while some authors still treat Chaos as a fashionable collection of new cocktail vocabulary, Chaos is neither a passing fad nor a mere metaphor. The extensive applications of Chaos to military systems make it **imperative** for today's decision makers to be familiar with the main results of the theory. This chapter is a quick review of the typical technologies where you should **expect** to see chaotic dynamics in military systems. The chapter is intentionally broad, since many more systems appear in Chapter Four, where we start to **apply** Chaos results. The present discussion concludes with a necessary review of the theory's limitations, as well as a summary of the implications of Chaos' pervasiveness.

Recall, from the previous chapter: **you don't need much** to generate chaotic dynamics. If a system changes continuously in time—like the motion of vehicles and missiles—you only need three independent variables (three degrees of freedom) and some nonlinearity. If your system changes in discrete jumps—daily aircraft sortie rates, or annual budget requests—then all you need is some nonlinearity, as simple as the squared

term in the logistic map. These minimum requirements, present in countless military systems, do not **guarantee** chaotic dynamics, but they **are necessary conditions**.

Other common characteristics that make a system **prone** to Chaos include delayed feedback and the presence of external perturbations, or “kicks”. There are enormous numbers of military systems with these features. You should **expect** Chaos in any system that includes feedback, fluids or flight. The power of Chaos Theory lies in its discovery of **universal** dynamics in such systems. As this chapter proceeds from specific systems to general technologies, be alert for the similarities in diverse military systems.

Naval Systems. Thompson’s text on nonlinear dynamics includes a thorough discussion of the chaotic behavior of a specific offshore structure.³³ He reports a case history in which chaotic motions were identified in a simple **model** of a mooring tower driven by steady ocean waves. Mooring towers are increasingly being used for loading oil products to tankers from deep offshore installations. These buoys are essentially inverted pendulums, pinned to the sea bed and standing vertically in still water, due to their own buoyancy. The concern in this “kicked” pendulum system is the potentially dangerous chaotic activity when a ship **impacts** the mooring. The number of impacts per cycle, which can be high, is an important factor to be considered in assessing possible damage to the vessel.

A 1992 Office of Naval Research report summarizes a series of studies identifying the sources of chaotic dynamics in other ocean structures: a taut multi-point cable mooring system, a single-anchor-leg articulated tower, an offshore component installation

system, and a free-standing offshore equipment system.³⁴ The author identifies key nonlinearities and analytically predicts transitions and stabilities of various structural responses. At the time of the report, experiments were still underway to verify the analysis. Ultimately, the analysis will suggest ways to **control these systems** better, and to enhance current numerical models for these systems.

The naval applications of Chaos Theory are not restricted, of course, to stationary structures. A recent graduate of the Naval Postgraduate School reports the use of nonlinear dynamics tools to control the motion of marine vehicles.³⁵ In this interesting application of Chaos results, the system itself does **not** display chaotic dynamics. However, the knowledge of **common transitions** away from stable behavior allows the author to **improve the trajectory control** of ships and underwater vehicles.

Information Warfare. As yet nebulously defined, the subdiscipline of military science tagged as Information Warfare certainly includes a number of electronic systems subject to chaotic behavior. In many instances, chaotic dynamics contribute to the design of entirely **new** systems with capabilities made possible by Chaos Theory. One enormous field of application is in the area of digital image compression. Simple equations that generate complicated **distributions** allow us to translate pictures into compact sets of instructions for reproducing those pictures.³⁶ By transmitting the **instructions** instead of the complete images, we can send thousands of times more information across the same transmission channels.

These **fractal image compression** techniques perform better on large images and color images than other current compression techniques.³⁷ In 1991, the **decompression** speed for the fractal method was already comparable to standard industry techniques. If this process doesn't become the new standard for real-time communication, it will probably drive the performance standards for other technology developments. This powerful technology is already making its way into military map making and transmission, as well as into real-time video links to the battlefield. Other potential applications arise in the next chapter.

Two additional features of electronic Information Warfare make it ripe for Chaos applications. First, the high-volume and high-speed of communication through computer networks include the best ingredients of a recipe for Chaos: modular processes undergoing endless iteration; frequent feedback in communications "handshaking"; frequencies (on many scales) faster than the time it takes many systems to recover between "events" (messages, transmissions, and backups). A second place to anticipate Chaos is anywhere the **digital** computer environment approximates the smooth dynamics of real systems. Many **iterated computations** have been shown to exhibit Chaos even though the associated physical systems do **not**.³⁸

Assembly Lines. A recent book on practical Chaos applications presents a detailed explanation of where to expect, and how to control, chaotic dynamics in automatic production lines.³⁹ The author focuses a few particular subsystems: vibratory feeding, part-orienting devices, random insertion mechanisms and stochastic (random) buffered

flows. Possible military applications include the automated control of systems such as robotic systems for aircraft stripping and painting, and automated search algorithms for hostile missiles or ground forces.

Let me conclude this introduction to chaotic military systems by recalling the list of technologies in the 1991 Department of Defense Critical Technologies Plan.⁴⁰ This time, though, we can note the most likely places where these technologies overlap with the results of Chaos Theory:

1. semiconductor materials & microelectronic circuits—they contain all kinds of nonlinear interactions; semiconductor lasers provide power to numerous laser systems whose operation can destabilize easily with any optical feedback into the semiconductor “pump” laser.
2. software engineering—refer to our discussion of Information Warfare, with feedback possible at unfathomable volumes and speeds.
3. high performance computing—see Items 1 and 2.
4. machine intelligence and robotics—require all sorts of control circuitry and feedback loops.
5. simulation and modeling—chaotic dynamics are being recognized in numerical models we’ve used for twenty years; look for more details in the next chapter.
6. photonics—laser and optical circuitry may be subject to Chaos at quantum and classical levels of dynamics.
7. sensitive radar—often combines the instabilities of electronics, optics, and feedback.
8. passive sensors—refer back to the night-light experiment!
9. signal and image processing—fractals allow new advances in image compression.

10. signature control—stealth technology, e.g., wake reduction in fluids.
11. weapon system environment—peek ahead to the next chapter's discussion of the nonlinear battlefield and **fire ant** warfare.
12. data fusion—attractors and Lyapunov exponents can summarize new information for military decision makers.
13. computational fluid dynamics—fluids tend to behave chaotically.
14. air breathing propulsion—engines consume fluids and move through other fluids.
15. pulsed power—power switching requires circuitry with fast feedback.
16. hypervelocity projectiles and propulsion—both areas include guidance, control and other feedback systems.
17. high energy density materials—can undergo chaotic phase transitions during manufacture.
18. composite materials—same manufacturing issues as Item 17.
19. superconductivity—superconductor arrays (Josephson junctions) are a classic source of Chaos.⁴¹
20. biotechnology—living organisms are full of fluids and electricity and . . . Chaos.
21. flexible manufacturing—may include automated processes prone to Chaos.

Limitations of Chaos Theory

It may seem difficult, after the previous section, to imagine **any** military system where we **won't** encounter Chaos. Let's do a brief reality check to indicate some systems that **do not** seem to benefit from the results of Chaos Theory. In general, Chaos will not

appear in slow systems, where events are infrequent, or where a great deal of friction dissipates energy and damps out disturbances.

For instance, we shouldn't expect Chaos Theory to help us drive a jeep or shoot a single artillery piece. On the other hand, the theory may eventually guide our decisions about how to direct **convoys of jeeps**, or how to space the timing or position of **many projectile firings**. Similarly, Chaos Theory offers no advice on how to fire a pistol, but it may help us design new rapid-fire weapons.

The theoretical Chaos results are seriously constrained by the need for **large amounts** of preliminary data. To make any analysis of time series, for instance, we **can** make reasonable comments based on as few as 100 data points, but the algorithms work best with about 1000 data points.⁴² Therefore, even if we are able to design reliable decision tools for battlefield use, if our models require input from 100 daily reports of enemy troop movements, we may be out of luck in a 30-day war. While I hold out some hope for the prospects of increasing our speed and volume of **simulated** battlefield information, the mechanisms for using such simulations for real-time combat decisions remain to be developed.

You may encounter scenarios and systems with erratic behavior, where a source of Chaos is not immediately evident. In this event, you may need to examine different **scales** of behavior. For example, Chaos Theory may not help study the flight of a **single** bird, free to choose where and when to fly. However, there is evidence of Chaos in how **groups** of birds flock and travel together!⁴³

Implications

The pervasiveness of chaotic dynamics in military systems forces us to be aware of sources of instability in our system designs. We need to develop capacities to protect our own systems from unwanted fluctuations, and to impose unwanted dynamics on enemy systems. However, the next chapter will also present ways we can constructively **exploit** chaotic dynamics, to allow new flexibility in control processes, fluid mixing, and vibration **reduction**. We must remain alert for new perspectives of old data that were previously dismissed as noise. Perhaps more important, the universal results of Chaos Theory open the door for new strategies—ideas we'll discuss in the chapter ahead.

FOUR

How Can We Use the Results?

Exploiting Chaos Theory

**One of the great surprises
to emerge from studies of nonlinear dynamics
has been the discovery that stable steady states
are the exception rather than the rule.**

- Gottfried Mayer-Kress⁴⁴

Introduction

You should have some intuition, at this point, for the common features of Chaos. You should also be comfortable with the fact that an enormous number of systems exhibit chaotic dynamics; many of these systems are relevant to military decision making. But how can we **use** Chaos to make **better** decisions or design **new** strategies? Even if we

accept the idea that Chaos **can** be applied to strategic thinking, shouldn't we leave this high-tech brainstorming to the analysts?

Absolutely not! As Gottfried Mayer-Kress points out, if we fail to learn the basic applications of Chaos Theory, our naiveté could lead to unfortunate consequences:

- the illusory belief that successful short-term management allows total control of a system;
- difficulty in making a diagnosis from available short-term data;
- application of inappropriate control mechanisms that can actually produce the opposite of the desired effect.⁴⁵

This chapter lays out practical results on how Chaos Theory influences a wide range of military affairs. Let's review what we've covered so far. In the introductory chapter, I suggested that Chaos provides us with new **information, courses of action and expectations**. The discussions in the first two chapters mainly focused on **expectation**: if we understand Chaos, then we are better able to interpret the behavior of systems we observe, and we have a greater appreciation for the kind of dynamics to expect in a system that can oscillate. Some of the previous examples foreshadowed the **new information** that Chaos results provide, and the **new options** available when we understand the consequences of the theory. Among many other things, Chaos Theory offers very specific dynamics and transitions to expect, tools to observe and assess, opportunities to prevent, induce, and control chaotic dynamics, **even (and especially!) in systems where there is no hope for modeling**. The sections that follow consolidate

these relevant consequences, with specific suggestions on **how to apply** these results. As you read through, the structure of each section may suggest that each concept or technique operates independently, like an isolated item in a tool kit. However, be alert to see how the application of Chaos Theory unifies many of the previous results.

This chapter begins with a review of some Chaos results that are consistent with past thought and with good common sense. The meat of this chapter, of course, is the discussion of the new tools and options available to decision makers because of the results of Chaos Theory. Then, an introduction to fractals begins a section on applications that take particular advantage of the fractal geometries that appear in many chaotic systems. Finally, we end with a discussion of other issues, including the difficulties posed by making decisions about systems that include human input and interactions.

Common Concerns

We should pause and consider the understandable concerns and objections of those who may be suspicious of “all this Chaos business.” It is quite easy to try and dismiss Chaos as an impractical metaphor, especially since many authors present **only** the metaphors of Chaos. Some toss around the Chaos vocabulary so casually that they leave us no hope for practical applications of the results. Margaret Wheatley, for one, only offers Chaos as a metaphor, hiding behind the argument that “. . . there are no recipes or formulae, no checklists or advice that describe ‘reality’ [precisely].”⁴⁶ While I certainly

agree that no formula can track individual trajectories in a sensitive chaotic system, especially with human choice involved, **many** patterns are evident, many means of observation and control are available, and the trends of chaotic dynamics are sufficiently common that we can and should expect specific classes of behaviors and transitions in chaotic systems. Unfortunately, even many well-written Chaos texts target a highly **technical** readership where the **useful** results are not adequately deciphered for a larger community of potential users.

All the same, we already **know** that human activity is sensitive to small disturbances, that small decisions today can have drastic consequences next week, and that troops—like water drops—need rest between events. It's simply not obvious there's anything **new** in the Chaos field. Why is it worth everybody's time just to learn a new vocabulary to describe the same old thing we've been doing for decades, or in some cases, centuries? Moreover, suppose we agree that there is something new here. How can we **use** the Chaos results? How can Chaos help me prioritize my budget or defeat my enemy?

Dr Peter Tarpgaard, a Naval War College professor in National Security Decision Making, offers a fine analogy to answer some of these concerns and to offer a glimpse of the insight that Chaos Theory brings to decision makers. To paraphrase, imagine what Galileo's contemporaries commented when they saw him depart for Pisa with a golf ball and a bowling ball in his duffel bag. "What's the use? You're gonna climb the Leaning Tower, and drop the things, and they're gonna fall. We **know** that already! You're not

showing us anything **new**. Besides, even if it is new, how can we **use** it?” What he found was qualitative evidence that objects with different properties fall at the same rate.

Now consider the tremendous advance in knowledge when **Newton** derived **precise expressions** for the force of gravity. Among other things, Newton’s laws of motion identified **specific behaviors to expect** when various objects are subject to gravity’s influence. By describing gravity’s effects, Newton gave us the power to model them—if only approximately—and to assess their impact on various systems. In particular, we now know exactly how fast an object will fall, and we can figure out when it will land. With this knowledge, we can also predict and control certain systems.

Chaos Theory brings comparable advances to decision makers. **The good news:** a number of researchers have developed techniques and tools that allow us to apply Chaos Theory in physical and human systems. The bad news: these efforts are very recent, and a great deal of thought and study remains to be done. This chapter represents my best effort to package my own results and those I have assembled thus far from available sources. Enormous research questions remain open; I outline some of these topics in the next chapters.

Something Old, Something New

Some of the consequences of Chaos Theory were recognized long before Lorenz uncovered the influence of nonlinearity in fluid dynamics. This lends some credibility to the results; as Clausewitz tells us, we need to compare new theories with past results to ensure their consistency and relevance. I've highlighted some examples below which come from standard topics in the Naval War College curriculum.

- Marine Corps Doctrine specifically discusses the advantage of getting “inside” your opponents’ OODA (Observe-Orient-Decide-Act) loops in order to increase their unpredictability (or “chance” or “fog” depending on your perspective). Similarly, in the U.S. Army Manual FM 100-5: “. . . in the attack, initiative implies never allowing the enemy to recover from the initial shock of the attack.”⁴⁷ This general strategy follows naturally from our observation of dripping faucets: Chaos results when the system is not allowed to relax between events.
- The US Marine Corps Manual FMFM-1 includes many references to the consequences of sensitivity to current states, and the unpredictability of plans and predictions:

We have already concluded that war is inherently disorderly, and we cannot expect to shape its terms with any sort of precision. We must not become slaves to a plan. Rather, we attempt to shape the general

conditions of war; we try to achieve a certain measure of ordered disorder. Examples include:

*[channeling] enemy movement in a desired direction,
blocking or delaying enemy reinforcements so that we can fight a
piecemealed enemy rather than a concentrated one,
shaping enemy expectations through deception so that we can
exploit those expectations,*

We should also try to shape events in such a way that allows us several options so that by the time the moment of encounter arrives we have not restricted ourselves to only one course of action.⁴⁸

- In a Naval War College lecture, Professor Michael Handel discussed the analysis of counterfactuals, alternative histories that might have occurred if key figures had made different choices. An important question: in a historical case study, how far can we carry our analysis of alternative strategies that were not actually pursued? His conclusion: the further ahead we consider, the less precision we should attempt to impose. In other words, the further we carry our counterfactual musings, the less reliable our analysis is.⁴⁹ This is an expression of sensitivity to initial conditions, correctly applied to historical analysis.

Consistent, At Least

We can see, then, that some of the consequences of Chaos Theory do **not** present new findings for strategic thought. However, we can be content that these preliminary observations are consistent either with our “common” sense or with the conclusions of previous researchers and thinkers.

So WHAT'S NEW ???

The applications presented in this chapter concentrate on methods, results, tools and traits of dynamical systems that were not recognized, or even feasible, only thirty years ago.

The fact that deceptively simple-looking functions and interactions can produce rich, complicated dynamics, constitutes a genuinely new insight. This insight grew from the work of biologists' simple population models like the logistic map that were analyzed in greater detail by mathematicians. As a result, we learned that complex dynamics and outcomes do **not** have to come from complex systems. Apparent randomness and distributions of behavior can be produced by very simple interactions and models. In particular, Edward Lorenz discovered that our difficulty in predicting weather (and many other chaotic systems) is not so much the resolution of the measurements as it is the vulnerability of the system itself to small perturbations. In fact, global weather is so sensitive, that even with a compact grid of satellites measuring atmospheric data at 1-kilometer increments, Prof Edward Teller estimates we could only improve our long-range weather forecasts from five days to a staggering 14 days!⁵⁰

So don't fire your meteorologists or your analysts! To simply expect and recognize Chaos in so many real systems is progress enough. The best news is that we have so many tools available to further understand and control chaotic systems. Think back to the aircraft loss data in Figure 2. The tools of Chaos Theory offer hope for

discerning the key processes that drive these erratic patterns. J.P. Crutchfield highlights the importance of nonlinearity in developing those tools:

[The] problem of nonlinear modeling is: Have we discovered something in our data or have we projected the new-found structure onto it? . . . The role of nonlinearity in all of this . . . is much more fundamental than simply providing an additional and more difficult exercise in building good models and formalizing what is seen. Rather it goes to the very heart of genuine discovery.⁵¹

Some amazing results discussed below include ways we can often **quantify** a system's sensitivity and give some estimate on **how long** predictions are valid. Only very recent advances in computers allow us to repeatedly measure quantities such as fractal dimensions, bifurcations, embeddings, phase spaces and attractors. The results of these measurements give us the information we need to apply the theoretical results. In this way, the tools of dynamical systems animate innumerable dynamics that have gone unobserved until now; decision makers who are aware of the tools available to them can better discern the behavior of military systems.⁵²

HOW TO APPLY

While the results of Chaos Theory offer us tremendous metaphors to improve our perspective of dynamics in military systems, the practical applications of Chaos go well beyond simple analogy. To highlight this point, you'll notice I pushed my discussion of Chaos metaphors to the end of this chapter. The chapter focuses, instead, on specific

processes, examples, and brief cases with suggested insights and uses for the analytical tools presented earlier. As you consider the applications of these results in your own systems, remember that sometimes you may **prefer** chaotic dynamics; at other times you want periodicity or stable steady states. In other instances, you may simply want to influence the unpredictability in a system: increasing it in your adversary's system, decreasing it in your own.

Feedback

The results of Chaos Theory help us to:

- **know what transitions to expect when we add feedback to a system;**
- **suggest ways to adjust feedback;**
- **appreciate the wide range of dynamics generated by feedback in real systems;**

There's nothing new about a call for awareness of feedback in physical and social systems. Many commentators, for instance, alert us to the impact of real-time media reporting combat events faster than DOD decision loops can operate. You may also consider the feedback imposed on your organization by requirements for meetings and reports. How often do you "pulse" your organization? Yearly, monthly, weekly, daily? Do you request periodic feedback, or do you allow it to filter up at will? Is the feedback

in your organization scheduled, formatted, free-flowing, “open door”, a mixture of these? How intense is this occasional “perturbation” to your system?

These are familiar issues for managers and commanders, but a grasp of chaotic dynamics leads us to answer these questions with other equally important questions. What mixture of structured feedback and freeform feedback works best in your system? What would happen if you increased or decreased the frequency of your meetings and reports? **What transitions in system performance should you expect?** Is it likely, for instance, that too many meetings of an office staff could generate instabilities in your system? Or, in a crisis situation—battle warfare, rescue, natural disaster—what characteristics of the “system” make it more appropriate to assess the system every day, or every hour? This idea was explored during a series of Naval War College war games. In these games, one out of every three messages was arbitrarily withheld from the commanders, without their knowledge. As a result, observers noted better overall performance of the students’ command and control processes.⁵³

Your awareness of your need for, and the sensitivity of, feedback in your system will make you more alert to the **possible consequences of altering the feedback**. The biggest benefit of Chaos Theory here seems to be **transitions we should expect** as we alter various parameters of system feedback.

For example, if you observe that too many meetings or reports cause undue stress on your organization, you might identify several obvious parameters under your control: frequency of feedback, length of reports, amount of detail or structure required in your reports, length of meetings, number of people involved in your meetings, etc. Some experience with dynamical systems suggests that small changes, or careful control, of

these parameters may suffice to stabilize some aspect of your system's performance. One new expectation we learn from chaotic systems is that small changes in control parameters can lead to disproportionate changes in behavior. Again, the idea of manipulating meeting schedules and reporting cycles is not new. However, the **expectations** for ranges of behavior and transitions between behaviors **are** new.

As a hypothetical illustration, suppose you observe changes in an adversary's behavior based on how often your surface vessels patrol near his territorial waters. Let's assume that your adversary bases no forces along the coast when you leave him alone, but he sets up temporary defenses when you make some show of force—say, an open water “Forward Patrol” exercise—once a year. Assume, further, that when you double the frequency of your exercises to twice a year, you note a substantial change in your adversary's behavior. Maybe he establishes permanent coastal defenses, or increases diplomatic and political pressures against you. You have cut the time difference between significant events (in this case, military exercises) in half and you observe a transition in the system. Now, it's an awful idea to pull Feigenbaum's constant out of our holster, fire it at this scenario, and predict that the next transition in the adversary's behavior will come if we decrease our time interval by only $(6 \text{ months})/(4.67)$, or 38.5 days. On the other hand, the common features of chaotic systems suggest that—even though we have no model for the system—we should at least be alert that the **next transition** in this system could come if we increase the frequency of our exercises by only a small amount, or it could come if we simply maintain the current semi-annual exercise schedule longer than the patience of the adversary can endure.

There may be few cases where we can afford the risk of testing such a hypothesis on a real adversary, though force-on-force dynamics like these **could** be simulated or gamed to reach significant, practical conclusions. You might consider, for instance, whether Saddam Hussein was playing a game just like this, in 1994, when he posted substantial forces along his border with Kuwait, while the United States military was busy with events in Haiti. Was he determining the increments of force size and timing that are necessary to provoke US military responses? I would guess that Hussein is not applying Chaos Theory to his strategic decisions, but **we** might analyze and game our **own** dynamics to see what increments of Iraqi force disposition will compel us to react. My hypothesis is that an understanding of chaotic dynamics ought to help us understand and control our response, selected from a flexible range of options, because knowledge of Chaos helps us understand the likely transitions when we change our system's control parameters.

Any one of the following additional questions would require a complete study in itself. However, I offer these ideas to stimulate your thought about the role of feedback, and transitions between behaviors, in systems that interest you:

- The increasing availability of real-time information to decision makers amplifies our concerns about information overload. How much detail does a leader require? How often? How much intelligence data does it take to saturate commanders and diminish their capacity for making effective decisions? What are the best ways to organize and channel a literal flood of information? The common transitions of chaotic systems

suggest that we might learn to control the flood by studying the effects of **incremental changes** in key parameters such as: volume of information, frequency of reports, number of sources involved in generating the data, and time allotted for decision making. Understanding the transitions from reasonable decision making to ineffective performance may help us tailor our intelligence fusion systems for the benefit of our commanders.

- The **relative timing** of an incursion on an adversary's decision cycle may be more important than the magnitude of the interruption. Many successful strategies hinge on "getting inside the decision cycle" of your enemy. The idea, of course, is to take some action, and then move with such agility as to make a subsequent move before your opponent has time to Orient-Observe-Decide-and-Act in response to your first action. Chaos Theory offers an important new insight to this basic strategy: we should expect ranges of different responses depending on how "tightly" we approach the duration of an OODA loop. That is, if we want to outpace an enemy who operates on a 24-hour decision cycle, we may find we have the same disrupting effect on his operations if we revise our Air Tasking Order every 18 hours as we would if we revise on a 12-hour or 6-hour cycle. We can then choose one planning option over another in order to meet other objectives for speed, economy of force, efficiency, increased monitoring of combat effectiveness, or resupply requirements. The idea is that we should expect ranges of control parameter values where the system behavior is relatively consistent; we should be prepared to note parameter ranges where small

adjustments translate into drastic changes in system response. Notice, this phenomenon is **not** sensitivity to initial conditions. Rather, it relates the sensitivity of the system structure and changes in **parameters**, or adjustments to the control knobs, if you will.

- One final application to consider, in another area of our decision cycle: coordinating our interactions with the news media during crises. We may find that we can adjust the time intervals of wartime press conferences, for example, to control the effects of real-time media feedback in our own decision loops, without having to resort to appeals for outright censure. Periodic feedback, carefully timed, could contribute to desired behaviors in domestic systems, like channels of public support, or adversarial systems who tune in to American television for intelligence updates.

Predictability

How does Chaos Theory explain, illuminate, reduce or increase predictability? Previous sections of this essay referred to the unpredictable nature of chaotic systems: the irregular patterns in dripping faucets, rocking boats, flickering lasers. Here, we'll consider the results that help us understand a chaotic system's erratic behavior. While the paths of individual chaotic trajectories can never be accurately predicted for very long, we'll see that knowledge of a system's attractors offers practical information about the long-term trends in system behavior. This section on predictions

begins with a summary of powerful results that also allow us to **predict the short-term behavior** of **chaotic** systems, even with no model! Then, this section on predictability concludes with an explanation of the usefulness of **attractors** for assessing **long-term system trends**.

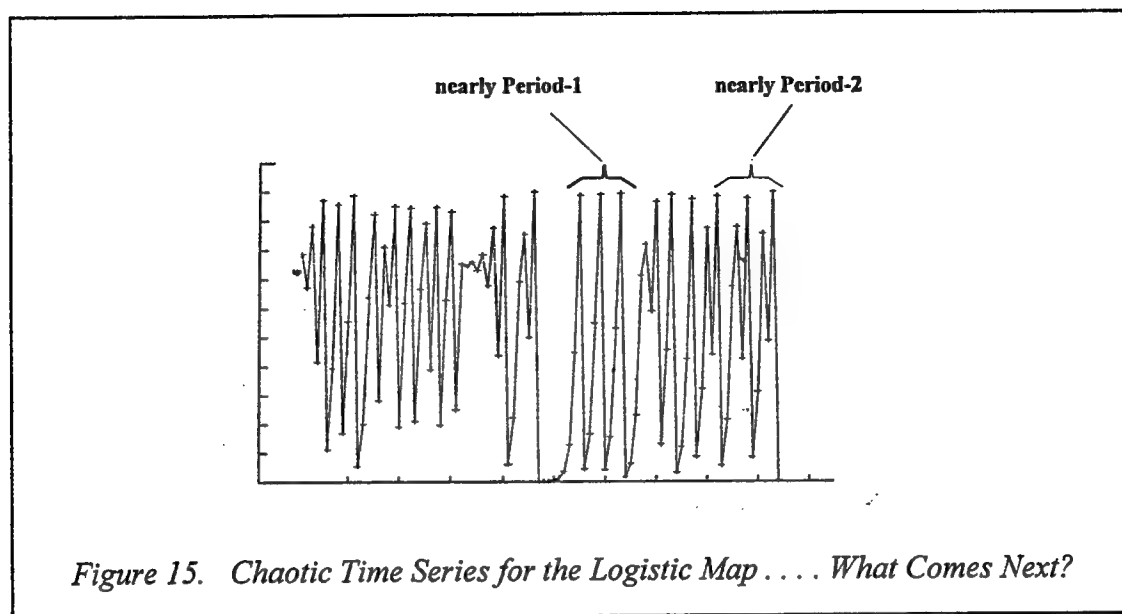
Time Series Predictions. We record—and sometimes analyze—unfathomable quantities of data at regular time intervals: daily closing levels of the Dow Jones Industrials; monthly inventory reports; annual defense expenditures. A list of measured data like this, along with some index of its time intervals, is called a **time series**. Simply visualize a long printout of numbers, organized in a table or graph, sequentially in time.

Now, if part of the list is missing, we might **interpolate** by various means to attempt to estimate the information we need. For instance, if we know a country's tank production was 30 vehicles three years ago, and 32 vehicles last year, we might guess that the production two years ago was around 30 tanks or so. To make this estimate we should first:

- feel confident in the data we have on hand;
- have some idea that industrial activity over the last few years was fairly constant;
- have some reason to believe the production cycle is annual and not biannual;
- perhaps, have access to a model that approximates this nation's production habits.

More often than not, though, we are concerned with **forecasting** issues, such as, how many tanks will that country produce **next year**? For such questions we must **extrapolate** and make some future prediction based on previous behavior. This is a perilous activity for any analyst, because the assumptions on which any models are made remain valid only within the time span of the original set of data. At any point in the future, all those assumptions may be useless.

Unfortunately, predictions of behaviors and probabilities are an essential activity for any military decision maker; we have to muddle through decisions on budgets, policies, strategies and operations with the best available information. Amazingly, however, the results of Chaos Theory provide a powerful **new means to predict the short-term behavior** of erratic time series that we would otherwise dismiss as completely random behavior! Very briefly, here is the basic idea. If you had a time series with an obvious pattern, 2 5 7 2 5 7 2 5 7 ... , you could probably predict the next entry in the list with some confidence. On the other hand, if you had a time series with erratic fluctuations, as in Figure 15, how could you know if there were discernible patterns to project into the future? Through the **embedding** process, Chaos analysts can uncover patterns and subpatterns that are not apparent to the naked eye, and use that information to project the near-term behavior of irregular dynamics like those depicted here. In Figure 15, for instance, you can see some evidence of behavior that approaches periodic behavior for a few cycles; embedding methods identify the places in phase space where these dynamics are most likely. This technique has been applied to several complex fluids and thermal systems with tremendous success.⁵⁴



The embedding technique, of course, does not work for all time series, and the predictions may only hold for a few cycles past the given data set. However, modern decision makers need to be aware of this tool for two crucial reasons. First—let’s be honest—without any hope from Chaos Theory, you and I wouldn’t dream of trying to predict a single step of the wild dynamics illustrated Figure 15. The theoretical results give us hope that we can now make **reasonable projections** in systems we previously dismissed as being beyond analysis. However, Figure 16 includes samples of the kind of predictions possible with embedding methods. Given 1000 data points from which to “learn” the system’s dynamics, the algorithm used here was able to predict fairly erratic fluctuations for as many as 200 additional time steps!

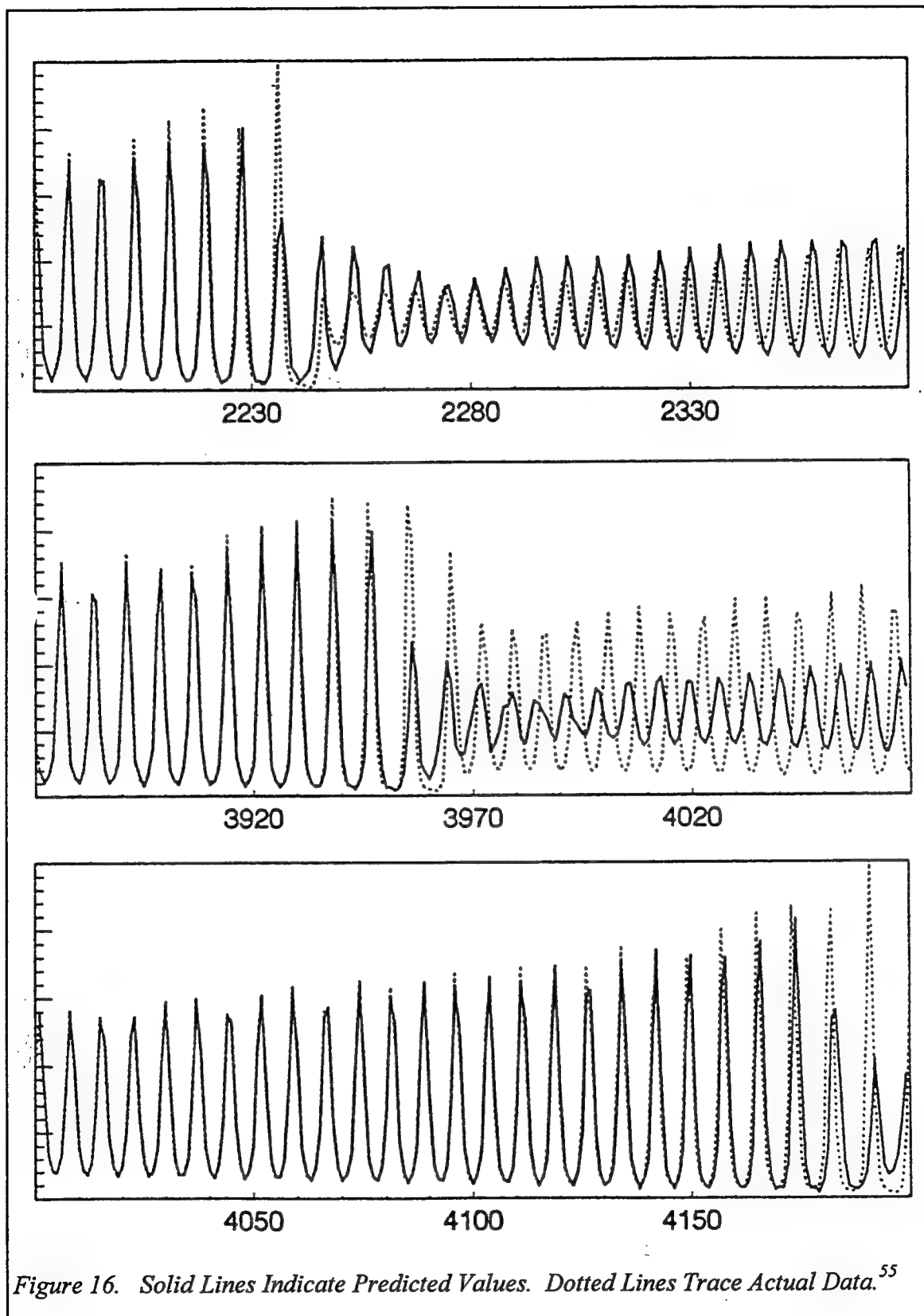


Figure 16. Solid Lines Indicate Predicted Values. Dotted Lines Trace Actual Data.⁵⁵

In addition, embedding methods include **estimates of the error** induced by extrapolating the data, giving the decision maker an idea of **how long** the projections may be useful! For detailed presentations of this technique, see, for instance, the notes from a 1992 summer workshop at the Sante Fe Institute.⁵⁶ Additional explanations also appear in a recent article by M. Casdagli, "Nonlinear Forecasting, Chaos and Statistics."⁵⁷ Both references outline the algorithms for near-term and global statistical predictions of chaotic time series. Still other researchers have successfully applied similar methods to enhance short-term predictions by separating background noise from chaotic signals; the list of distinguished authors includes Ott, Sauer and Yorke,⁵⁸ J.D. Farmer,⁵⁹ and William Taylor of the RAND Corporation.⁶⁰

Attractors and Trends. I can not overemphasize: the sensitive character of chaotic dynamics denies us any hope of predicting the long-term behavior of a system, regardless of how accurately we can measure its current state. On the other hand, any knowledge of a system's **attractors** gives us plenty of useful information to predict long-term trends in the system. Just look outside. Without the benefit of a meteorology degree you can probably tell whether or not you'll need an umbrella to cross the street. You may even have enough information to make reasonable short-term decisions—like if you should go to the park this afternoon—even though the **long-term** weather remains unpredictable. On a larger scale, you can tell the difference in how to pack for a vacation in Hawaii, versus a trip to Moscow, without **any** current weather information at all.⁶¹ And don't

forget how fortunate we are that the weather behaves chaotically and not randomly. Otherwise, we'd have no hope of making even short-term forecasts!

These simple examples illustrate how we make decisions based on some knowledge of system **trends**. The attractors of a dynamical system provide precisely that information. Whether we construct an attractor from measured data, or from extensive simulations, a system's attractor can **illustrate trends** that are not as intuitive as the simple weather examples above. Moreover, a well-drawn picture of an attractor vividly displays the relative amount of time the system spends in certain regions of its phase space. Now, up to this point, this kind of information was available even before the advent of Chaos Theory.

However, Chaos Theory brings us several new results when we are confident an erratic system is truly chaotic. First of all, by simply recognizing an attractor, we regain some hope that we can understand and manipulate our system. After all, the attractor gives form and structure to behavior we used to dismiss as random. J.M. Thompson points out in his Nonlinear Dynamics text:

*analysts and experimentalists should be vitally aware that such apparently random non-periodic outputs may be the correct answer, and should not be attributed to bad technique and assigned to the wastepaper basket, as has undoubtedly happened in the past. They should familiarize themselves with the techniques presented here for positively identifying a genuine chaotic attractor.*⁶²

Many practical pieces of information can be derived from our knowledge of a system's attractor. First of all, the relative amount of time the system spends on various portions of the attractor constitutes a **probability distribution**; an attractor could provide

key probability information to a military decision maker in many scenarios. Secondly, if we find an attractor for a system, then even if we keep the system parameters constant, any disturbances to the system's current state will still render its particular evolution unpredictable (envision a tire-swing, or a vibrating space station). However, any transient behaviors **must** die out and the **global trends** of system behavior must be unchanged. After all, that's what the attractor describes: regions of phase space that attract system dynamics. Third, we have some hope that we can predict or recognize the **basins of attraction** in a given system.⁶³ If we can prepare a battlefield or a negotiation scenario to our liking, we have some hope we can set up its initial state so the system proceeds under its own dynamics to the trends of the attractor we desire.

Visualization of attractors also makes system **transitions** more apparent as we change control parameters. Recall, for instance, the return maps we sketched for the dripping faucet (Figure 6). It's important to notice that, when the period-2 behavior **first** occurs, the pair of points in the attractor "break off" from where the single point used to be. A bifurcation occurs here; we find that the **periods** of these **initial** period-2 cycles are **very close** to the previous period-1 time intervals. Thus, by **tracking the attractors** for various parameter settings, we not only observe the individual dynamics, but also discern **additional information about the transitions** between those behaviors.

Unfortunately, most real dynamical systems are not simple enough to collapse onto a single attractor in phase space. How can we understand and exploit **multiple attractors** in a single system? Here's an analogy: when my '85 Chevette starts up in the morning, it warms up at a relatively fast idle speed. This is one periodic (non-chaotic) attractor for the operation of my car engine with some fixed set of parameters. A few

minutes later, when I tap accelerator to release the choke, the engine idles, but much more slowly. The system output has fallen onto a second periodic attractor. The system is the same, but an external perturbation “bumped” the system to a new, bounded, collection of states.

My friend CDR Millward now asks, is there any chance of exploiting the existence and **proximity** of two attractors in a system? Say that our system of interest is the disposition of an enemy force, and suppose the current set of control parameters allows that system to evolve along either of two attractors, one of which is more to our own advantage. By examining the control parameters available to us, is it possible we **manipulate the transitions** between these attractors, joining them, breaking them, building or destroying links between them? These questions may, at first glance, appear too metaphorical, but as our facility with models and intelligence data increases, we may find that the answers to these questions bring extremely practical strategies to the table.

Here’s a brief summary of the practical guidance Chaos Theory offers for system **predictability**:

1. Techniques like **embedding** make short-term prediction possible in chaotic systems.
2. These techniques **quantify the short-term reliability** of a given forecast.
3. **Attractors** describe the **long-term recurrent behavior** of a system.
4. The relative time spent in various states on the attractor defines useful **probabilities**.

5. Images of attractors give indicators of the **features of system transitions**.
6. The presence of **multiple attractors** may provide us new ideas for strategic options.

Control of Chaos

In this section, we study one of the most powerful consequences of Chaos Theory: we can take a chaotic system—whose behavior we used to dismiss as random—and drive out the Chaos to impose stability! Moreover, this is often **possible without the aid of any underlying model!** This capability has no counterpart in nonchaotic systems. You may be amazed to see the numerous real systems where researchers successfully controlled the behavior of chaotic systems.

To lend some structure to the list of examples that follows, you should look for three basic approaches that have been demonstrated for Chaos control:

1. regular periodic disturbances;
2. proportional inputs, based on real-time feedback;
3. trajectory “steering,” based on models or approximations to the dynamics on an attractor.

We already tried the first control technique when we induced periodic output in the chaotic dripping faucet by tapping a rhythm on the spout. In some regards, this technique is consistent with standard results of resonance theory that describe how external vibrations can excite certain natural frequencies in the system. However, in a

chaotic system we are **guaranteed** to find that **infinitely many different periodic behaviors** are possible, not just combinations of the natural modes of system.

The second control method, on the other hand, requires real-time measurements of the system's output in order to determine how far to adjust the selected control parameter. This is a generalization of the way you would balance a long stick on the palm of your hand: you move your hand just enough, based on how you feel the stick leaning, and you manage to keep the stick upright. This method has the disadvantage of requiring a reliable feedback-driven control loop. The obvious advantage, though, is that we achieve stable output **intentionally**; not in the hit-and-miss fashion that sometimes characterizes control experiments of the first type.

The third control method was recently developed at the Massachusetts Institute of Technology (MIT). It requires extensive calculations in order to develop approximations to the dynamics on a system's attractor. It has not been reported in any other experiments yet, but I include it to provide a peek at new results to come.

These three techniques are the most practical means available to **control** systems that would otherwise exhibit Chaos; the methods allow us to impose different types of stability, depending on the application. For example, the stability you generate **may** be a stable steady state (like balancing the stick), but it may also be a stable periodic state (often desirable in laser systems). It's also possible to **eliminate** the possibility of Chaos entirely by modifying the system in some way (see the discussion below on **process**). However, in this section, we'll look at Chaos **control**, ways to lock on to one of the infinitely many unstable periodic trajectories densely woven on an attractor.

The key observation common to all three techniques is that a chaotic attractor typically has, kneaded into it, an infinite number of unstable periodic orbits. Here are the main benefits we derive from **Chaos control**:⁶⁴

- We can convert Chaos into one of many possible attracting periodic motions by making only **small perturbations** of an available system parameter.
- The method uses information from previous system dynamics, so it can be applied to experimental [real-world] **situations in which no model is available for the system.**
- **Control becomes possible** where otherwise large and costly alterations to system may be unacceptable or impossible.

Several **good references** describe the complete analytical details needed to implement these control algorithms:

- Ott, Grebogi and Yorke perfected the technique that uses real-time feedback; current publications refer to this method with the authors' initials, the "OGY method."⁶⁵ Since their initial report, they (and many others!) have applied the OGY method to numerous systems, from classic chaotic systems like Lorenz' weather model and the logistic map, to physical systems such as thermal convection loops, cardiac rhythms and lasers. For example, Figure 17 shows the stable steady state imposed on the logistic map, compared to its usual, irregular dynamics.⁶⁶ The OGY team has also applied this method of Chaos control **to reduce and filter noise** that is present in measured data.⁶⁷
- The other control technique which is computation-intensive was developed by Elizabeth Bradley at MIT.⁶⁸ Like the OGY Method, her approach actively exploits chaotic behavior to accomplish otherwise impossible control tasks. Bradley's method, though, is more like a numerical interpolation. She successfully demonstrated her method on the Lorenz equations. Though it is not yet fully automated, and requires a tremendous amount of data or a complete model, the technique shows great promise.

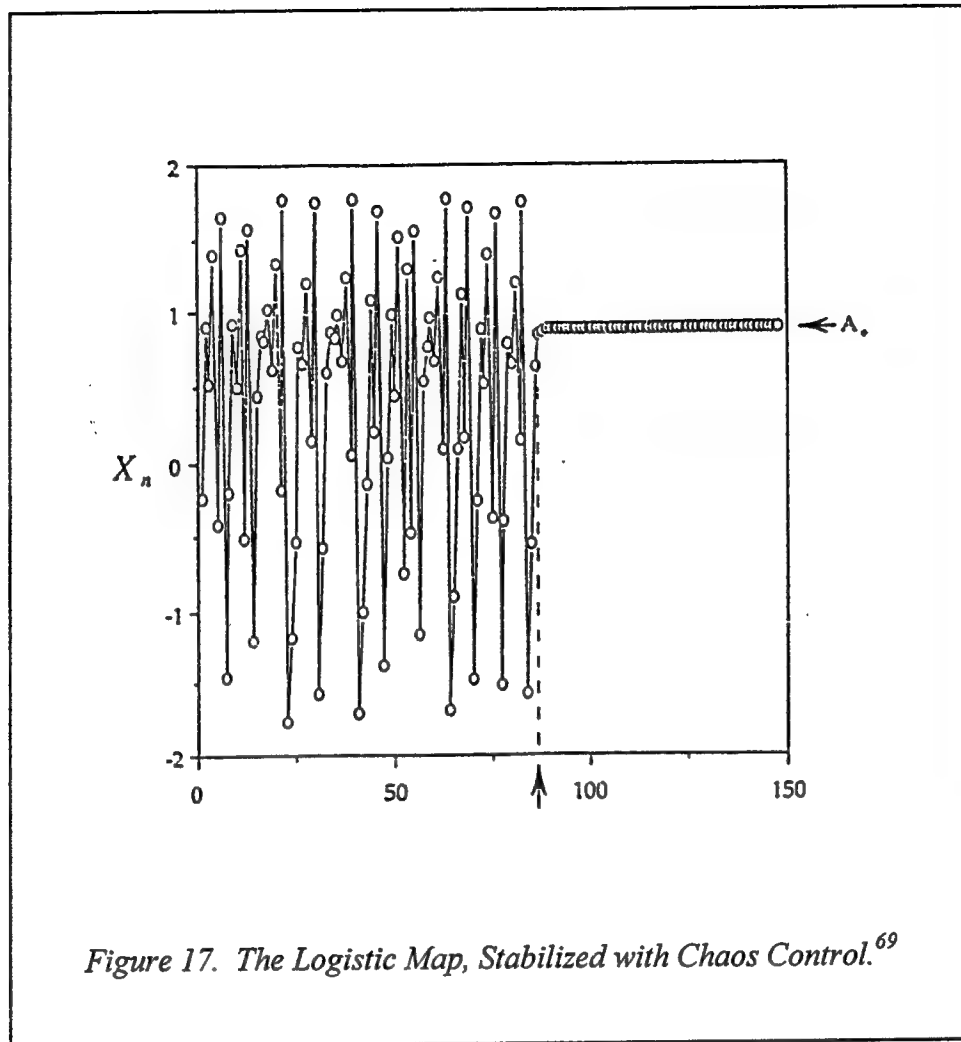


Figure 17. The Logistic Map, Stabilized with Chaos Control.⁶⁹

Applications of Chaos Control

Thin Metal Strip. Early applications of the OGY method stabilized vibrations in a thin metal strip. Based on real-time measurements of the strip's position, the apparatus automatically adjusts the frequency and amplitude of input vibrations. This simple experiment confirmed validity of Chaos control theory, stabilizing period-1 and period-2 behavior, and switching between the two at will. These early successes highlighted the important consequences of Chaos control:

- no model was needed,
- minimal computations were required,
- parameter adjustments were quite small,
- different periodic behaviors were stabilized for the same system,
- control was possible even with feedback based on imprecise measurements.⁷⁰

Most important, this method is clearly not restricted to idealized laboratory systems!!!

Engine Vibrations. Henry Abarbanel summarizes the results of several vibration control studies for beams, railroads, and automobiles.⁷¹ He describes the use of automated software to discover the domains of regular and irregular motions in beams driven by external vibrations. This information is important to the study of lateral railroad vibration, known as **hunting**, which deforms and destroys railroad beds. The hunting phenomenon—recognized for decades, but never traced to its source—was shown to arise through the same period doubling transitions we saw in our dripping faucet and the logistic map! Understanding the source of these oscillations should lead to ways of mitigating the vibrations, saving tremendous costs in safety and maintenance. In another case, a vibration absorber for rotating machinery (S.W. Shaw, University of Michigan) successfully removed unwanted oscillations by prescribing paths for counterrotating dynamical elements. The induced motions precisely cancel vibrations in helicopter and automotive machinery. You can expect these nonlinear absorbers to appear soon in products of the Ford motor company, who sponsored the work.

Helicopter Vibrations. Chaos Theory was recently applied, for the first time, to study flight test data from OH-6A Higher Harmonic Control (HHC) test aircraft. The HHC is an active control system used to suppress helicopter vibrations. Most vibrations in the

system are periodic, but evidence of Chaos was found. The presence of Chaos **limits** the ability to predict and control vibrations using conventional, active control systems, but, here, control techniques take advantage of the chaotic dynamics. Like the simple metal strip experiment, this approach uses only experimental data—no models!! By extracting information from time series, we can find the limits of possible vibration reduction, determine the best control mode for the controlling system, and get vibrations under control using only a few minutes of flight data. These powerful analytical results reduced flight test requirements for the HHC; the same methods can be applied to other vibration control systems.⁷²

Mixing. A South Korean company builds washing machines that reportedly exploit Chaos Theory to produce irregular oscillations in the water, leading to cleaner, less tangled clothes.⁷³ Whether or not we believe this particular claim, we ought to consider military systems where effective mixing might be enhanced by Chaos control, for example, in the combustion of fuel vapors in various engines.

By now, you should notice common features in the dynamics of many different systems, so you shouldn't be surprised to see comparable results in lasers. . . .

Flickering Laser. In a low-power laser at the Georgia Institute of Technology, Prof Raj Roy controlled the chaotic output of a laser by manipulating the laser's power source. Very slight, but periodic, modulations of the input power forced the laser into similar periodicity.⁷⁴ In this case, Chaos control was possible **without the use of feedback**. While the laser output was not driven to any specific target behavior, repeatable

transitions were observed, from Chaos to periodicity, when Roy modulated a single control parameter.

Chaos control also finds a number of applications in circuits and signals. . . .

Ciphers. In cryptography, as well as in many simulation applications, we need to produce large lists of pseudorandom numbers, quickly and with specific statistical features. Chaotic dynamical systems appear to offer an interesting alternative to creating number lists like these, though sometimes more work is necessary.⁷⁵ Unfortunately, the same embedding techniques that allow us to make short-term predictions of chaotic behavior also **limit** the use of chaotic systems for generating random-looking sequences. However, Chaos has other applications for secure communications. . . .

Synchronized Circuits. Even the simplest circuits can exhibit sensitive, unpredictable long-term chaotic behavior. However, with the correct amount of feedback, two different circuits can be synchronized to output **identical chaotic signals**. This extraordinary result could prove useful for securing communications by synchronizing chaotic transmitters and receivers.⁷⁶

Taming Chaotic Circuits. Elizabeth Bradley, at MIT, has completed software that takes a differential equation, a control parameter, and a target point in phase space, and approximates the system dynamics in order to drive a trajectory to a desired target point.⁷⁷ While computationally intensive, her approach has had good success controlling the Chaos in nonlinear electrical circuits. This technique takes information about dynamics on the attractor and translates that information into approximate dynamics that allow **control of individual trajectories**. As a result, this technique provides a more global view of control processes.

Human systems? I have not yet seen Chaos **control** knowingly attempted on social systems, but consider, for instance, the options available for controlling the periodic dissemination of information to decision makers, both friendly and adversary. On the operational and tactical scales, we can envision many ways to apply periodic perturbations to a combat environment through action, inaction, deception, and information control. From a more strategic perspective, consider how regular negotiations and diplomatic overtures can tend to stabilize international relations—measures whose absence can allow relations to degenerate unpredictably. Depending on how you **define such a system**, you might observe truly chaotic dynamics and new opportunities to control those dynamics. Of course, I temper my optimism by emphasizing that active human participants can adapt unpredictably to their environments. However, a discussion follows shortly on the evidence of Chaos in human systems, offering some hope for applications.

Remember the big idea:

IF, a system is known to be (potentially) chaotic,

THEN its attractor must contain an infinite number of unstable periodic trajectories.

The presence of all those densely packed periodicities makes Chaos control possible.

There are further implications for **system design**, since it's possible not only to modify a chaotic system **very efficiently**, with small control inputs, but also to choose from a **range of desired stable behaviors**. Therefore, novel system designs are possible: we may be able to design a **single system** to perform in several **dissimilar modes**—like a guided weapon with several selectable detonation schemes, or a communications node

with diverse options for information flow control. Current designs of systems like these often require us to build parallel components, or entire duplicate systems, in order to have this kind of flexibility. However, knowing that Chaos is **controllable**, we can now consider new system designs with **Chaos built in**, so that various stable behaviors can be elicited from the exact same system through small, efficient perturbations of a few control parameters!⁷⁸

Chaos and Models

Why bother with applying Chaos to modeling? Some of the following concerns are common to any debate about the utility of modeling:

- To increase the doctrine's emphasis on the **human** aspects of war, Air Force Manual (AFM) 1-1 argues in detail that war must not be treated like an engineering project.⁷⁹
- There will always be trade-offs between the detail you'd like in the model and the detail you really **need**. Gleick summarizes nicely: "Only the most naive scientist believes that the perfect model is the one that perfectly represents reality. Such a model would have the same drawbacks as a map as large and detailed as the city it represents, a map depicting every park, every street, every building, every tree, every pothole, every inhabitant, and every map. . . . Mapmakers highlight such features as their clients choose."⁸⁰
- Sometimes, even when good models are available, initial states can not be known (regardless of considerations of precision). For example, what initial conditions should be assumed for a complex model of the atmosphere, or an oilrig at sea in a developing storm? . . . How can we hope to explore the responses from all possible starts?⁸¹

Sensitivity to Initial Conditions (SIC), of course, brings into question whether there is any utility at all in trying to run a computer model of a chaotic system. Why bother, if we **know** that **any** initial condition we start with must be an approximation of reality, and that SIC will render that error exponentially influential on our results as we move forward in time? Wheatley, among others, maintains a grim outlook on the whole modeling business in the face of SIC.⁸² Jim Yorke, however, has **proven** that even though a numerical chaotic trajectory will **never** be exactly the trajectory we want, it **will** be arbitrarily close to some **real** trajectory actually exhibited by the model itself.⁸³

Here are a few more of the many reasons we should struggle to understand the role of Chaos in modeling and simulation:

- **Practical Dimensions:** The calculation of a time series' fractal dimensions is a means of assessing the number of effective independent variables determining the long-term behavior of a motion.⁸⁴
- **Counterintuitive Outcomes Prevail:** Simple computer models can be used to study general trends and counterintuitive consequences of decisions that otherwise appear to be good solutions. The results of even simple models will broaden our perspective of what **can** occur, as much as what is **likely** to occur.⁸⁵
- **Attractors Depict Trends:** Chaos results can help validate the behavior of models whose output appears erratic. When we can't match the individual time series, we can often match the distribution of behavior on the entire attractor.

Chaos in the Simplest Models. Even a brief survey of recent military models will reveal the **importance of expecting Chaos** in models and simulation. Ralph Abraham, for instance, gives a detailed analysis of what happens in his model of opinion formation. His numerical exploration is a good demonstration of the process of wringing out a

model. Chaos appears as he models the interaction of two hostile nations responding to the relative political influence of various social subgroups.⁸⁶ Other researchers at Oak Ridge National Laboratory have demonstrated a range of dynamical behavior, including Chaos, in a unique, competitive combat model derived from differential equations.⁸⁷

Recent RAND research has uncovered certain classes of combat models that behave much like chaotic pendula. The authors discovered chaotic behavior in the outcomes of a very simple computerized combat model. Preliminary results offer ideas to better understand nonintuitive results and to improve the behavior of combat models.⁸⁸ For example, war game scenarios often produce situations where an **added capability** to one side leads to a **less-favorable result** for that side. Results like these have often been dismissed as coding errors. The correct insight, of course, is that non-monotonic behavior is caused by nonlinear interactions in the model. In the simple RAND model, reinforcement decisions were based on the state of the battle, and the resulting nonlinearities led to chaotic behavior in the system's output. The RAND team drew some interesting conclusions from their simulations:

- While models may not be predictive of outcomes, they are useful for understanding **changes** of outcomes based on incremental adjustments to control parameters.
- **Scripting** the addition of battlefield reinforcements (i.e., basing their input on time only, not on the state of the battle) **eliminated** chaotic behavior. This may not be a realistic combat option, but it's valuable information regarding the battle's dynamics.
- The authors were able to identify the **input parameters figuring most importantly** in the behavior of the non-monotonicities: in this case, the **size** of the reinforcement blocks and the **total number** of reinforcements available to each side.

- **Lyapunov exponents** were useful to evaluate the model's sensitivity to perturbation.

In general, the RAND report concludes, “for an important class of realistic combat phenomena—decisions based on the state of the battle—we have shown that modeling this behavior can introduce nonlinearities that lead to chaotic behavior in the dynamics of computerized combat models.”⁸⁹

Dockery and Woodcock, in their detailed book, *The Military Landscape*, provide an exceptionally thorough analysis of several models and their consequences, viewed through the lenses of catastrophe theory and Chaos. New perspectives of combat dynamics and international competition arise through extensive discussions of strategy, posturing, and negotiation scenarios. They uncover chaotic dynamics in classic Lanchester equations for battlefield combat with reinforcements. They also demonstrate the use of many Chaos tools, such as Lyapunov exponents, fractals, and embedding.⁹⁰

Dockery and Woodcock appeal to early models of population dynamics—predator-prey models—to model interactions between military and insurgent forces. The predator-prey problem is a classic demonstration of chaotic dynamics; the authors use common features of this model to simulate the recruitment, disaffection and tactical control of insurgents. The analogy goes a long way and eventually leads to interesting strategic and tactical conclusions, illustrating:

- conditions that tend to result in periodic oscillation of insurgent force sizes;
- effects of a limited pool of individuals available for recruitment;
- various conditions that lead to steady state, sustained stable oscillations, and chaotic fluctuations in force sizes;

- the extreme sensitivity of simulated force strengths to small changes in the **rates** of recruitment, disaffection and combat attrition.

In one of the many in-depth cases presented in *The Military Landscape*, patterns of dynamics in the simulation suggest candidate strategies to counter the strengths of insurgent forces. The model is admittedly crude and operates in isolation since it can not account for the adaptability of human actors. However, the model does point to some non-intuitive strategies worth considering. For example, cyclic oscillations in the relative strengths of national and insurgent forces can result in recurring periods where the government forces are weak while the insurgents are at their peak strength (Figure 18). If the government finds itself at this relative disadvantage, and adds **too many** additional resources to strengthen its own forces, the model indicates that the cyclic behavior tends to become unstable (due to added opportunities for disaffected troops to join the insurgent camps) and paradoxically weakens the government's position. Instead, the chaotic model's behavior suggests carrying out moderately low levels of military or security activity to **contain the insurgents at their peak strength**, and await the weak point in **their** cycle before attempting all-out attacks to destroy the insurgents' forces completely.⁹¹

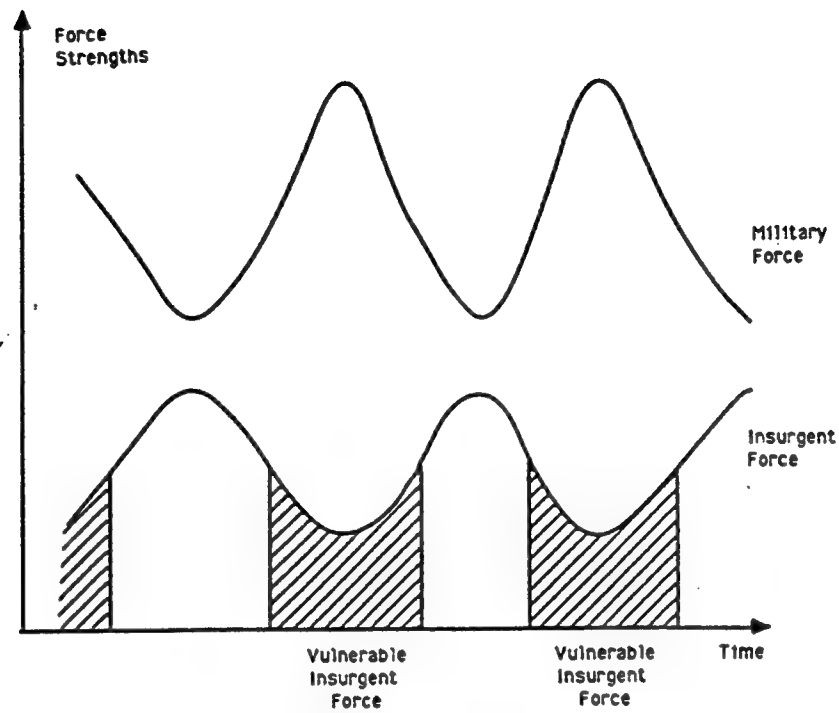


Figure 18. Periodic Changes in Force Levels in Two-Force Simulation.⁹²

Process

Since many approaches to Chaos Theory remain uncharted, we often find, in reports of experiments as well as in analyses, that the **processes** followed are as instructional as the results. The laser system I studied at Georgia Tech with Prof Raj Roy is a good example.⁹³ We started with a low-power laser whose output intensity fluctuated irregularly when we inserted a particular optical crystal into the cavity. The crystal converts a portion of the available infrared light into a visible green beam which is useful

for many practical applications. Even though a previous set of equations described **some** of the laser's operation, no one had yet discovered the **source** of the fluctuations. Alternating between output from numerical models and the real laser, we modified the model, using reasonable basic physics, until our **numerical** results displayed Chaos. As a result, we identified the specific source of chaos, and we were able to eliminate the chaotic fluctuations. This is one approach you might consider for analyzing a system if your **system** exhibits Chaos, but the **model** you're working with does **not**.

If you find your model behaves chaotically, but the real system doesn't, you have a few options. You may, of course, simply have fundamental mistakes in your model. However, a more subtle possibility is that you may simply need to **reduce one of your parameter values** (i.e., decrease the "energy" in the model) until the model matches reality. Option three: if you're confident in the model, **be alert** for conditions when the real system might have different parameters. **EXPECT CHAOS!**

If both the system and its model show Chaos, you should (at least) compare attractors, the distributions of the measurable output, like the histogram we drew in Chapter Two. Are the bounds on the attractors comparable? Do the densities of points on the attractors correspond? Once you gain confidence in the model, try to draw explicit connections from **model parameters** to quantities you can measure **in the system**. This is how to get control of the Chaos in your system.

These approaches have many potential applications, such as **generating distributions** for use in war-gaming models. If we can replace random algorithms in war game models, with simple chaotic equations that produce comparable distributions, we should find clues leading to the parameters that play the greatest role in the dynamics of given scenarios.

Exploit Chaos for Strategies and Decisions

What's new about the application of Chaos results to strategic thinking? In general, our awareness of the new possibilities of **how** systems can behave brings us definite advantages. Sometimes we will **want Chaos**. Perhaps an adversary's system will be easier to defeat if it is somehow destabilized. Cryptologists may prefer chaotic dynamics to secure their communications. On the other hand, many systems function better in **stable, periodic conditions**: signal transmissions, long-range laser sensors, and regular, predictable international relations. Fortunately, Chaos Theory also teaches us new ways to assure **system stability** through careful control of feedback.

Alan Saperstein pinpoints several new ideas that Chaos Theory brings to the strategic planner:

- Many previous attempts to analyze international relations included notions of stability and instability that are **not** new in the Chaos results. However, previous models do not account for or produce **extreme sensitivity to small changes** in input or model parameters.
- Models have proven to be very useful for identifying trends, transitions, and parameter ranges where stability or instability is prevalent.
- It follows that, if incomplete models of international conflict show instability in given regions of parameter space, then more complete, "realistic" models are also likely to be unstable in larger regions of the parameter space, i.e., harder to stabilize.
- The converse is **not** true: if a given model, representing a system, is stable, then a more complex, more realistic model of the same system may still be unstable.⁹⁴

The ideas in this section overlap somewhat with the previous sections on Chaos applications. The focus, though, is to assemble specific insights, options and techniques available to military decision makers and strategic planners. The examples proceed from specific results to general approaches. Look for connections among the many efforts to apply Chaos Theory to military activities.

Decision Making Tools. Here is a concise summary of analysis tools, developed in the study of chaotic dynamics, available to military decision makers. These tools have surfaced throughout previous chapters in various examples and discussions:

- Given sufficient data, time series analysis allows us to make short-term predictions, even in chaotic systems.
- Lyapunov exponents help us quantify the limits of our predictions and measure our system's sensitivity to small disturbances. This information can help to prioritize various strategic options according the relative unpredictability of their outcomes.
- Knowledge of common transitions in chaotic systems can suggest ideas for protecting and attacking military systems.
- Calculations of attractors depict distributions of outcomes, providing probability information to decision makers.
- Calculations of information dimension indicate the minimum number of variables needed to model a system. Moreover, a small value for dimension also represents strong evidence that the underlying dynamics are **not** random. A system with a non-integer dimension **must** contain nonlinearities (i.e. any previous models that are strictly linear must be incomplete).⁹⁵

Pattern Recognition. In recent research at the Air Force Institute of Technology, the theory of **embedded time series** allowed Capt James Stright to automate the process of **identifying military vehicles** from a few measurements of vehicle position and velocity. He also determined **how long** a data sequence we need in order to accurately classify these moving objects. You can visualize the basic concept: the position of a drone aircraft with locked controls, for instance, should be far easier to predict than the position of a piloted aircraft conducting evasive maneuvers. So Capt Stright generalized the idea of tracking objects as they move. At any fixed time, he notes a vehicle's position and velocity and logs that information in a vector. Evolutions of these vectors comprise an embedded time series; the patterns evident in this embedding allow us to characterize **typical** vehicle behaviors. Capt Stright verified his technique, correctly distinguishing the motions of five military vehicles.⁹⁶

Feedback Revisited. Earlier in this chapter, we discussed the role of feedback in chaotic military systems. Chaos Theory brings new insights and options to strategies that include “pinging” an enemy system to see how it responds. Think, first, about the various parameters we can control when we perturb an adversary's system—a large ground force, for instance. We can strike it periodically or unpredictably. We can change the magnitude (firepower), character (area versus directed fire), and frequency of our assaults. We can attempt to induce or reduce chaotic responses. We can reduce the amount of feedback in the system through operations security and information control. You might also envision particular attack strategies that apply our study of night-light dynamics to long range perturbations of various sensors present on enemy systems.

Again, suppose we are forced to close a base or a port, and replace our “Forward Presence” there with a “Forward Patrol” or “Frequent Exercise” of some itinerant military presence. Chaos Theory highlights relevant parameters we should consider in our strategic planning, to include: the size of patrolling forces, the distances to the areas of

interest, and the frequency of patrolling activities. Further, the dynamics common to chaotic systems give us **specific transitions** we might expect in our adversary's response as we vary any of those key parameters.

Fire Ants. Chaos applications in future strategies will follow in the wake of numerous revolutions in military technology. One such revolution may come in the form of "fire ant" warfare—combat of the small and the many. It will see a battlefield covered with millions of sensors (the size of bottle caps), emitters (like pencils), microbots (like mobile computer chips), and micro-missiles (like soda bottles). These many devices will be deployed by a combination of pre-positioning, burial, air drops, artillery rounds, or missiles, and will saturate regions of the battlefield terrain.⁹⁷ The need for us to understand the dynamics of weather systems and clouds suddenly becomes more than an academic exercise, because "fire ant" warfare produces a new combat climate: battlefields filled with new clouds that carry lethal capabilities. Anyone designing an enormous autonomous system like this, with millions of nonlinear interactions, better be familiar with the complete range of possible dynamics, as well as with the means to control and defeat such a system.

SDI Policy. Saperstein describes another use of Chaos in a numerical model to guide policy and strategy. His paper carefully qualifies his findings in an intelligent, numerical exploration and appropriately cautious use of modeling. The policy question was whether or not an implementation of a Strategic Defense Initiative would tend to destabilize an arms race between two superpowers. In this case, he relied on a nonlinear model to predict the outcomes of various options to help guide the policy making. Saperstein emphasizes his model is a procurement model, not a force-on-force simulation, that includes inventories and production rates of various types of weapons. Among his conclusions, he finds, for example, that a bigger qualitative change in the

opponent's behavior comes with the introduction of **defensive weapons**, more so than with even drastic increases in annual ICBM production. Also, beyond his specific findings, his work exemplifies the **process** of using models to guide decision making.⁹⁸

Operational Art. In the Joint Military Operations Course at the Naval War College, we often focus on the following four questions facing the operational artist:

1. What military condition must be produced in the theater of operations to achieve the strategic goal?
2. What sequence of actions is most likely to produce that condition?
3. How should the resources of the force be applied to accomplish the desired sequence of actions?
4. What are the costs and risks of performing that sequence of actions?

The operational commander, of course, has access to the same tools available to any decision maker. Using these tools, the most direct applications of Chaos results are likely to be in answers to Question 2, where Chaos tools can provide information about **probabilities of outcomes**. Notice, too, that when we provide probability information to a commander, the quantity and frequency of our intelligence analysis also represent **feedback** in our decision processes, feedback that can produce transitions in our own performance.

The second most likely use of Chaos will come in the answer to Question 4, where we balance the costs and benefits of various courses of action. This paper proposes the use of Lyapunov exponents to help us prioritize our options based on the relative unpredictability of our actions. No simulations or computer programs have yet been developed to implement this idea.

Finally, Chaos Theory may also address issues raised in Question 3, developing options for force application, when one of the following conditions holds:

- We have access to enough **well-synthesized data** on our adversaries' behavior to allow us to make near-term predictions of their actions.
- The opponents use **sensors** or electronics that allow us to control their systems through feedback techniques.
- We face a **large force**, where we can exploit our knowledge of the distribution of behaviors in large interacting systems.
- We engage in **prolonged combat**, to have sufficient time for our observations of enemy behavior to reveal trends and patterns in enemy responses.

Exploiting Chaos. Overall, we need to anticipate chaotic dynamics so we can exploit them in our own systems as well as in enemy systems. A final caveat: besides the necessary reminder that combat participants can adapt in surprising ways, also remember that **unpredictable changes** in enemy dispositions can also turn in the **enemy's** favor. In 1941, for instance, Japan managed to destabilize America's isolationist position by bombing Pearl Harbor. The fact that this destabilization worked against their hopes underscores the problem that the uncertainty produced by **arbitrary disruption** can lead to many unpredictable results, sometimes for better, sometimes for worse. Fortunately, the results of Chaos Theory discussed above offer many strategic options beyond the mere disruption of enemy systems.

Information Warfare Revisited

Earlier we noted the vulnerability of communications systems to Chaos. Vast numbers of coupled electric systems, many of which are controlled with feedback mechanisms, process unfathomable quantities of information, all at the speed of light,

with frequent iterations. Without the details of a given system, we can't **guarantee** the onset of Chaos, but we should definitely **expect** chaotic dynamics in systems with the above characteristics.

So far, we've mentioned the potential implications of enhanced data compression for Information Warfare, and the need to be aware of the numerical Chaos sometimes present in digital computations. I mention Information Warfare again in this section to tie together a few other applications discussed above. For one, Chaos applications in secure communications, in encryption and in synchronized circuits, will certainly play a part in Information Warfare. Also, Capt Stright's automated algorithm for pattern recognition could eventually be applied to identify information "targets" as well as it identifies physical targets.

Fractals

Fractals have many more applications than merely serving as identifiers for time series with non-integer dimensions. Fractals play important roles in system scaling and in other image compression applications. First, we'll examine some consequences of the multiple scales of dynamics present in real systems. Then we'll see how researchers take advantage of these multiple scales to compress images with fractal transformations.

Scaling. We can gain new perspectives of military systems by considering dynamics on various physical scales, scales that become evident through the study of fractals. For

instance, you can probably see Chaos right now, in a system somewhere near where you're reading this: in the traffic patterns outside your building, in a stop sign wobbling in the wind, in the light flickering overhead or on your computer display. However, likely as not, there are many nearby chaotic dynamics occurring on physical scales that you probably don't care about: quantum fluctuations, or irregularities in the power output from your watch battery. The important idea is that you may sometime encounter system behavior you can't explain because there may be key nonlinearities on a scale you haven't considered yet.

Once we develop an awareness of the **universality** of many chaotic dynamics, we realize that some dynamics and physical properties occur on **all** scales in many systems, both natural and artificial. Gleick expresses this idea quite eloquently, guiding us to cases where we should expect to see scale-independent structures and dynamics:

How big is it? How long does it last? These are the most basic questions a scientist can ask about a thing. . . . They suggest that size and duration, qualities that depend on scale, are qualities with meaning, qualities that can help describe an object or classify it. . . .

The physics of earthquake behavior is mostly independent of scale. A large earthquake is just a scaled-up version of a small earthquake. That distinguishes earthquakes from animals, for example—a ten-inch animal must be structured quite differently from a one-inch animal, and a hundred-inch animal needs a different architecture still, if its bones are not to snap under the increased mass. Clouds, on the other hand, are scaling phenomena like earthquakes. Their characteristic irregularity—describable in terms of fractal dimension—changes not at all as they are observed on different scales. . . . Indeed, analysis of satellite pictures has shown an invariant fractal dimension in clouds observed from hundreds of miles away.⁹⁹

Many other common systems will exhibit the same dynamics on virtually any scale: hurricanes, fluid flow, airplane wings and ship propellers, wind tunnel experiments, storms, and blood vessels, to name only a few.

How does our awareness of scaling properties broaden our perspective of military affairs? Just like we can conserve time and money by experimenting with scale models, we can sometimes resolve questions about a system's behavior by examining one of its components on a more accessible scale. For example, the electronic architectures of our war game facilities nationwide are being configured to network as many sites as possible to conduct large-scale simulations. Unfortunately, the combat dynamics that are simulated at different facilities operate on different **scales of combat**, where some are tactical simulations, some operational, and others strategic. War game designers are currently faced with difficult questions concerning how to connect the flow of information among these participants on differing scales. The answer may eventually lie in a network based on fractal scaling of some kind.¹⁰⁰

Fractal Image Compression. The need for data compression grows more apparent daily as ships at sea saturate their available communication links, and users worldwide crowd a limited number of satellites and frequency bands.¹⁰¹ Other requirements for information compression arise in large modeling problems, where physicists, for example, try to model cloud dynamics in simulations of laser propagation. One recent breakthrough in image compression came from Michael Barnsley's ingenious manipulation of fractals, leading to a process defined in his Collage Theorem.¹⁰²

To compress an image of a leaf, for instance, Barnsley makes several smaller copies of the original image, and covers the original with the smaller copies. He tabulates all the **transformations** necessary to shrink, rotate, and translate those copies in order to cover the original leaf. That list of transformations is the only information necessary to reproduce the original image! Now, rather than transmit a picture of a leaf via pixel-by-pixel arrays of hue and brightness, we can **transmit a brief set of instructions** so the receiver can **redraw** the leaf very efficiently. By transmitting these short instruction sets, Barnsley's process compresses large color images in excess of 250:1. Not only has Barnsley demonstrated this process with simple images, but he's proven that **you can derive transformations for any image, up to the best resolution of your sensor!**

The tremendous compression ratios by these fractal compression techniques make possible new applications in digitized maps for numerous uses, including devices for digitized battlefield equipment and avionics displays. Moreover, the end product of this transmission process is, in fact, an attractor of a chaotic system, so it contains density information about how often a given pixel is illuminated by the receiver's redrawing program. Among other uses, this local density information translates into useful data for the physicist interested in propagating lasers through clouds.

Barnsley's company, Iterated Systems, Inc., has already had several Army and Navy research contracts to make further advances with this compression technique.¹⁰³ One of the resulting products was a patented algorithm for pattern recognition, with the potential to develop automated means to prioritize multiple target images for a weapons system. Iterated Systems has also used fractal compression to transmit **live motion video** across **standard telephone lines**, a capability with numerous operational applications.¹⁰⁴

Metaphor

**You don't see something
until you have the right metaphor to let you perceive it.**

- Robert Shaw ¹⁰⁵

I've made this section deliberately short. Chaos does offer powerful metaphors that lend genuinely new perspectives to military affairs, but we have access to so many practical applications that flow from Chaos Theory, I will minimize this brief digression. The main idea is that the metaphors of Chaos bring a fresh perspective—not just a new vocabulary for old ideas. This perspective comes with an awareness of new possibilities: new information (fractal dimensions, Lyapunov exponents), new actions (feedback options, Chaos control), and new expectations (stability, instability, transitions to Chaos).

In a recent attempt to use Chaos metaphors for new historical perspectives, Lt Col Theodore Mueller, at the Army War College, depicts the Mayaguez crisis as the result of a system destabilized due to its sensitivity to small disturbances. He uses the image of an attractor to describe departures from the “range of expected behavior” for an adversary.¹⁰⁶ In another case, a Sante Fe Institute Study generalizes the results of classic predator-prey equations and draws interesting politico-military analogies from simple models. The

study makes a rough comparison of how the onset of epidemics, modeled in these equations, compares to social dynamics that may spark political revolutions.¹⁰⁷ More case studies applying Chaos metaphors are likely to follow, as the military community grows familiar with the theory's more practical results.

The Human Element--Chance, Choice and Chaos

Problems. Certainly, Chaos Theory can boast a tremendous record in mechanical and numerical applications, but can we, and should we, use these results in systems that include human input? How do we reconcile Chaos results with the apparently random dynamics of unpredictable human decisions, the transient nature of social systems, or the Clauswitzian interaction of adversaries in combat?

Some of these questions necessarily arise in **any** debate over the utility of **modeling** a system that includes human decisions or responses. In particular, we have cause for suspicion, because the analysis of social systems assumes we are able to **recognize and predict** trends in human behavior. If such predictions are possible, where does that leave our perspective of choice and free will?

Even if we suspend our disbelief long enough to explore candidate models for human behavior, we face significant obstacles to executing our analysis:

- Aggregate data sufficient for strong empirical tests simply do not exist for many important social systems.
- Social systems are not easily isolated from their environment.

- Social systems encompass huge scales in time and space, vast numbers of actors, cost variables, and ethical influences.
- The laws of human behavior are not as stable as the laws of physics.¹⁰⁸

This section argues that Chaos Theory does shed light on human behavior that is relevant to military affairs. Certainly, Chaos is only one of the many rich dynamics we can observe in human behavior. However, I will focus on some of the constraints on human behavior that give us reason to hope for some insight from chaotic modeling and simulation efforts. Next, I'll present recent evidence of the presence of Chaos in human behavior. Finally, I'll offer some initial ideas on how we might apply additional Chaos results to military affairs.

Hope. Let's look at some sources of hope for understanding human systems with the help of Chaos Theory. First of all, despite our seemingly unlimited capacity for creativity, we will always make decisions always within constraints imposed by limited resources, limited time, personal habit, and external pressures such as policy and opinion. Some of our constraints stem from periodic cycles in our environment, both natural and fabricated:

- 24-hour days
- human physical endurance
- seasonal changes
- planetary motion

- tides
- revisit times for a satellite with a small footprint
- equipment reliability and maintenance
- replenishment and resupply
- time cycles necessary to conduct battle damage assessment
- budget cycles
- periodic elections

This list is not intended, of course, to promote astrology applications in strategic planning. However, we've seen plenty of physical examples where **periodic perturbations can drastically alter a system's dynamics**, causing significant shifts toward or away from stable behavior. My position is that the pervasiveness of these constraints, often periodic constraints, gives us hope to **expect chaotic dynamics** even in systems influenced by human decisions and responses.

Another reason to be optimistic about Chaos applications in human behavior comes from the very nature of attractors: within an attractor's basin, transient behavior will die out and a system will only be found in states that lie on the attractor. Even if we perturb the system at a later time, it must return to the attractor. Now, I'll present some evidence below which points to the existence of non-random chaotic dynamics in human systems. Those dynamics, in turn, imply the presence of attractors for those systems. This does **not** imply that there is no influence of choice and chance in these systems.

Rather, I submit that, in these cases, **human decisions** represent one of the following influences:

1. **perturbations** of behavior which would otherwise remain on an attractor,
2. **changes in the distributions** of behavior, i.e., tendencies of the system to stay on any particular portion of the attractor, or
3. **choices from multiple attractors** that exist in a single system.

My guess is that we will eventually find phase spaces with **multiple attractors** to serve as the model for the various options available to us or to an adversary. As a playful analogy, think about the possible “state” of your mind as you read this essay; suppose we can somehow characterize that state by measuring your thoughts. Is there any hope of controlling or manipulating that system? If you think not, consider what happens to your thoughts when I tell you, “DON’T think of a pink elephant”? Whatever attractor your mind was wandering on before, did your thoughts pass through my “pink elephant” attractor, even momentarily? I contend that we have hope of modeling, understanding, and perhaps controlling some features of human influences in military affairs, perhaps only briefly, but long enough to enhance the planning and execution of numerous military activities from acquisition to combat.

In a study of two species of ants, where social dynamics are much easier to observe in a controlled environment, Nobel Prize winners Nicolis and Prigogine give us some additional hope for making analyses of human systems:

*What is most striking in many insect societies is the existence of two scales: one at the level of the individual, characterized by a pronounced probabilistic behavior, and another at the level of the society as a whole, where, despite the inefficiency and unpredictability of the individuals, coherent patterns characteristic of the species develop at the scale of an entire colony.*¹⁰⁹

While they draw no premature conclusions about the immediate consequences of these results for human behavior, they offer this evidence a reason to be optimistic about the possibility of analyzing and controlling group dynamics. Ralph Abraham, of the University of Michigan, also reminds us that we can study human decisions through game theory, where chaotic dynamics have already surfaced in the conduct of different games. A number of complex models are already making significant progress in explaining the actions and reactions among multiple players.¹¹⁰

Evidence of Chaos. Is there **evidence** of chaotic behavior in human systems? Let's remember what sort of symptoms we're looking for: a well-defined system, a clear list of observables to measure, aperiodic changes in those observables, bounded output, sensitivity to small disturbances, evidence or knowledge of nonlinear forces or interactions, attractors with fractal dimension, and small, non-integer information dimension. Several research papers report findings of many of these symptoms in **historical data** as well as in **simulations** using models that correspond well with observed human behavior.

Robert Axelrod, for one, has created a model that predicts how elements in a system group themselves into patterns of compatible and incompatible elements. He modeled nonlinear interactions with basins of attraction that predict how multiple actors

in a scenario will form opposing alliances. Typical aggregation problems where his results may apply include: international alignments and treaties, alliances of business firms, coalitions of political parties in parliaments, social networks, and social cleavages in democracies and organizational structures. The basic inputs to his model are a set of actors, the size of each nation-actor, their propensity to cooperate with each other, partitions (physical and otherwise), the distance between each pair, and a measure of “frustration” (how well a given configuration satisfies the propensities of a country to be near or far from each other actor). **Axelrod’s theory correctly predicts the alignment of nations prior to World War II**, with the exception that Poland and Portugal were mistakenly placed on the German side. He also had comparable success predicting how computer businesses would align behind various market standards, such as the selection of operating systems. His prediction correctly accounted for 97% of the total number of firms in the sample.¹¹¹

In another discovery of Chaos in social systems, Diana Richards presents several examples of experimental and empirical evidence in strategic decision making. First, she expands a simulated prisoner’s dilemma game, to illustrate possible dynamics in collective decision making in politics and economics. In this model, nonlinear interactions arise because the players’ decisions depend on their responses to actions in previous steps. She allows each of two simulated participants to choose from 100 options; various stable and chaotic dynamics result when she iterates the model.

On one hand, Richards emphasizes the difficulties in verifying such a model, because of the problem of collecting real data over as many repetitions as she can easily simulate numerically. On the other hand, she was able to apply time series analysis to

uncover chaotic dynamics in historical data. In particular, she discovered evidence of Chaos in US defense spending (as percentage of total federal spending) between 1885-1985, and in the number of written communications per day (between and within governments) during the Cuban missile crisis, October to January 1962.¹¹² **Again, the presence of Chaos in these systems does not indicate that their behavior is completely predictable, but the number of variables which drive their dynamics may be much smaller than our intuition might suggest, and we may have a better chance of modeling, understanding, and controlling these situations than previously thought possible.**

A tremendous study of historical data was completed by a team of students at the Air Command and Staff College (ACSC) in 1994. I found their report to be the most thorough research to date which examines historical data with the tools of Chaos Theory. Their calculations of fractal dimensions and return maps present conclusive evidence of Chaos in tactical, operational and strategic dynamics of military activity:

- aircraft loss data for the entire Vietnam War (See Figure 2);
- Allied casualty data during their advance through western Europe in World War II;
- historical US defense spending (results consistent with the Richards report, above).¹¹³

Recent investigations of well-known models in system dynamics have revealed previously unsuspected regimes of deterministic Chaos. One outstanding example is

John Sterman's comparison of two numerical models to controlled tests with human players. The first scenario is a production-distribution model of the Beer Distribution Game, where subjects are asked to manage a product inventory in the face of losses, delays in acquiring new units, multiple feedbacks, and other environmental disturbances. Despite the difficulties of conducting controlled experiments, Sterman found that the **human subjects' behavior is described fairly well by the model dynamics**. This direct experimental evidence that **Chaos can be produced by the decision-making behavior of real people** has important implications for the formulation, analysis, and testing of models of human behavior.¹¹⁴

Sterman's second scenario simulates a long economic wave in which players adjust inventory orders in response to long-term indicators of supply and demand. The simulated business begins in equilibrium; an **optimal** response to the provided indicators actually returns the system to equilibrium within six annual cycles. However, of the 49 subjects tested, none discovered the optimal behavior, and the **vast majority of subjects produced significant oscillations, many of which showed evidence of Chaos**.¹¹⁵

Further practical evidence of Chaos in individual behavior is discussed in recent NASA-sponsored research. In lab tests, researchers take measurements of a human EEG in efforts to characterize the "error prone state" of, say, a tired pilot. Are some individuals more prone to enter these states than others? What is the EEG signature of such a "hazardous state of awareness"? They found that standard statistical tools could not distinguish the EEG signal of an individual engaged in various activities, from mental math to image identification. However, the average **pointwise (fractal) dimension** of the EEG **did distinguish** the different types of activity. This work has the potential to

develop automated monitoring of pilots in flight, to warn of levels of decreased alertness. More generally, this gives us more hope for applying Chaos results to understand the dynamics of human behavior.¹¹⁶

Implications. There are still very few documented attempts to apply Chaos results to social systems, partly due to the newness of Chaos Theory, and partly due to the practical problems discussed above. However, many authors have noted important **implications** of the evidence of Chaos in social systems. Hal Gregersen and Lee Sailer, for instance, draw the following conclusions:

- Social studies rely too much on single measurements of population cross-sections; we need to focus instead on data taken incrementally over long periods of time.
- We need to recognize Chaos and use the new tools of dynamical systems in addition to standard statistical analysis.¹¹⁷

The ACSC research team also offers a good summary of the implications of chaotic dynamics in the data they studied:

- Many erratic systems are at least partly deterministic, so don't throw out your noisy data.
- The presence of Chaos requires models to include nonlinear interactions.
- The inclusion of nonlinearity implies that models will likely have no analytical solution, so don't throw out your computers (or your analysts)!

- Fractal dimensions estimate minimum number of variables needed to build models.
- Some regions of phase space are more sensitive than others; chaos tools can help identify those different regions.
- Tracking the patterns in attractors also helps identify excluded regions of behavior.¹¹⁸

How To Apply the Results. Ultimately, we will need to verify any theoretical claims in comparisons with real systems. In light of the problems of matching numerical models to human behavior, we're left with two basic options:

1. construct and analyze formal models only, comparing model results to historical data;
2. develop lab experiments with human subjects interacting with computer-simulated social systems, or "microworlds."¹¹⁹

These two options still leave us a lot of room to apply Chaos Theory to the study of social systems. For instance, Gottfried Mayer-Kress sets up a simple model of a superpower arms race, and discusses several immediate consequences of his simulated results. Surprisingly, the model gives little-to-no warning of the onset of political instability via the usual transitions to Chaos.¹²⁰ Thus, the use of a chaotic model can **indicate uncommon transitions to unstable behaviors**, providing new insight to what **can** happen in reality, despite the crudeness of the model.

How might we specifically adapt Chaos results to **organizational behavior**? A recent article discusses The Conference Model™, a series of conferences structured to help a large group implement effective reorganization. The process involves several carefully structured steps that involve a large number of group members, to encourage “ownership” of the process, comparable to current DOD Total Quality policies and processes. The authors report significant success with their process; it can be couched in terms of Chaos Theory to shed light on outcomes to expect from their suggestions for further research.

To begin, they define their system well: basically, an organization with fixed membership, divided into subgroups of managers and employees, planners and doers. The key parameters are the number of people of the various groups involved in the planning activities, the number of meetings, the number and timing of follow up activities. The measures of effectiveness include the time required to design the organization’s plan for change and the time taken to implement the changes.

One of the issues raised in this study: what is the **outside limit** on the number of people who can attend a conference? I would recast this question as an issue about the **ranges of possible dynamics**, as any of the key parameters are changed. For instance, what **transitions** are likely as the number of participants involved in the planning process decreases gradually from 100% of the organization? At what point do we note a substantial decrease in the effectiveness of the plan’s implementation? The universal results of chaotic dynamics suggest we should expect specific transitions (e.g.,

oscillations of some type) sometime before we reach the point of total failure of the planning process.¹²¹

John Sterman's conclusions about his lab experiments provide a good summary of the tremendous potential, and the unresolved issues, of applying Chaos to human systems:

- Test results show that **participants' behaviors can be modeled** with a high degree of accuracy by time-tested decision rules.
- **New chaotic dynamics have been noted**, in well-accepted models, for reasonable parameter ranges.
- The evidence strengthens the arguments for the **universality of these phenomena**.
- The **short time scales** of important social phenomena often render the utility of Chaos questionable.
- **The role of learning** is difficult to gauge: e.g., in the experiments discussed here, thousands of cycles are simulated, however, evidence shows that subjects began learning after only a few cycles.¹²²

Most important, the results demonstrate the feasibility of subjecting theories of human behavior to experimental test, in spite of the practical difficulties. Chaotic dynamics will continue to surface in future investigations; we need to be prepared to expect and recognize those dynamics when they occur.

Chaos and Military Art

This chapter compiles substantial evidence of predictable, controllable dynamics governing many aspects of military affairs. Am I saying there is no room left for Military Art? Quite the contrary, while chaotic dynamics are sufficiently universal to revolutionize our profession, Chaos Theory is only one of many necessary tools. Where is the individual art of the commander still evident? A good simulation, for instance, or a good summary of intelligence estimates, may draw a clear picture of an adversary's attractor. Maybe the image displays trends in force deployment, in aircraft ground tracks or in satellite footprints. However, an **attractor only helps express probabilities** to the commander. The commander still requires a sense of operational art to **evaluate those probabilities** in various courses of action, **assess the risks** of diverse options, and **choose** a single course of action.

What Do You Want Us To DO?

This nontrivial question was posed by a concerned audience member after I presented an introduction to Chaos at ACSC. I'm convinced we must **not** leave Chaos to the analysts, and wait a few years for more results. I encourage you to gain confidence that you can learn the essential material from good readings and patient thought. You can discern good sources from bad, using the Chaos Con tips and good sense. You can build more intuition for what to expect, what Chaos can do for you, when you need to consult

your in-house analysts, when you need to pay a contractor to do more research, and when you should tell the contractors to go to the library and do their own homework on their own money. You should develop an expectation, an anticipation, for chaotic dynamics in the motion and changes you observe daily.

Be confident reading. When you write, use the vocabulary with care, and **at least**, avoid the pitfalls I outline in my section on the Chaos Con! However, **DO WRITE**. Publish your progress and successful problem-solving and models, to show others your process for applying the results of Chaos Theory. Above all, be aware of the avenues open to you due to the **far-reaching results** of Chaos Theory.

David Andersen outlines several additional points he feels should be highlighted when we teach anyone about chaotic dynamics. These points certainly offer good advice for any decision maker considering the application of Chaos to military affairs:¹²³

- Understand phase plots in order to develop an intuition for Chaos.
- Learn to distinguish between transient and steady-state dynamics.
- Be ready to spend time computing.
- Take the time to get some theoretical background.
- Learn to recognize when Chaos might be near and how to diagnose it when it appears.

Chapter Summary

Tremendous opportunities await us in the numerous realms of Chaos applications.

We have access to new insights and strategic options that were inconceivable only 20 years ago: universal transitions in system behavior through the careful control of system feedback; new capabilities to predict short-term dynamics and long-term trends; options for controlling erratic systems previously dismissed as random; extraordinary advances in computations that enhance our communications capacity and improve our simulations. In the end, despite reasonable concerns about the utility of modeling, in general—and the analysis of human systems, in particular—we find a wealth of new information, actions and expectations made possible due to the continuing advances made in the understanding of Chaos Theory.

PART III

What Next?

A Roadmap to More Chaos

FIVE

Suggestions for Further Reading

This chapter summarizes the best resources I encountered during my research. Many Chaos books have appeared in just the last four years; this review only scratches the surface of the fantastic pool of published resources, not to mention numerous videos and software. My aim is to offer some guidance: to instructors on sources to recommend for additional reading; to students on the best leads for more detail; to all readers curious about the individuals and organizations who are researching and writing in diverse areas.

The focus of my essay has been to build a bridge from Chaos Theory to **your** areas of interest; I selected the following books and periodicals to serve as a rough map of interesting destinations for you to consider. The most thorough, well-developed readings came from: Gottfried Mayer-Kress (numerous articles), Woodcock and Dockery (*The Military Landscape*), John D. Sterman (writing in a special issue of *System Dynamics Review*, which you ought to read), James Gleick's classic, *Chaos*, and a special issue of *Naval Research Reviews* devoted to Chaos research sponsored by the Office of Naval Research. Further discussion of these and other references follows.

James Gleick, *Chaos: Making a New Science* (New York: Viking Penguin Inc, 1987).

Gleick composes vivid descriptions of the people and places at the roots of Chaos Theory. He interlaces his narratives with detailed personal interviews. His book is very readable, and he assumes no technical background. This book is not the best place to learn the **details** of Chaos—the concepts presented are very general—but it's a pleasant exposition of the wonder of discovery, the universality of Chaos, and its range of applications. Take the time to read all the endnotes where Gleick hides additional interesting facts. A great piece of storytelling.

Heinz-Otto Peitgen, Hartmut Jürgens and Dietmar Saupe, *Chaos and Fractals: New Frontiers of Science* (New York: Springer-Verlag, 1992).

The authors have compiled a veritable encyclopedia of Chaos. The text is very readable, assumes very little technical background, and explains fascinating connections among diverse Chaos applications. If you only put one Chaos book on your shelf, you should consider this one.

***System Dynamics Review* 4 (nos. 1-2, 1988).**

This special issue assembles a fine collection of articles which discuss important issues of Chaos Theory in great depth. The topics range from the very practical to the philosophical. John D. Sterman, for instance, opens the issue with a well-written introduction that surveys the basic concepts and results of Chaos Theory; he also contributes a strong paper on "Deterministic Chaos in Models of Human Behavior: Methodological issues and Experimental Results." This is another **must-read** resource.

J.M.T. Thompson and H.B. Stewart, *Nonlinear Dynamics and Chaos* (NY: John Wiley & Sons Ltd., 1986).

The authors aim this superb text at engineers and scientists, analysts and experimentalists. They require as background only "a little familiarity with simple differential equations." Step by step, they introduce Chaos, what to expect, and how to interpret data sets with irregular behavior; they use plenty of helpful pictures and graphs. In addition, they present a healthy range of applications, focusing on the ways simple models can generate complicated dynamics in: slender, vibrating structures, resonances of off-shore oil production facilities, large-scale atmospheric dynamics, particle accelerators, chemical kinetics, heartbeat and nerve impulses, and animal population dynamics. They also include a fantastic bibliography with over 400 entries. This is a great book from which to learn Chaos Theory.

John T. Dockery and A.E.R. Woodcock, *The Military Landscape* (Woodhead Publishing Limited, Cambridge, England, 1993).

This book presents an exceptionally detailed analysis of several models and the implications of their dynamics, viewed through the lenses of catastrophe theory and Chaos. New perspectives of combat dynamics and international competition surface during the analysis of the models' behaviors. The authors discuss extensive applications in strategy, posturing, and negotiation. In one of their many simulations, they uncover chaotic dynamics in the classic Lanchester equations for force-on-force combat, with reinforcements. They demonstrate the use of many Chaos tools, and they take great pains to show the **relationships** among the tools. Overall, this book includes more analytical details than most recent reports, and it is a thorough review of many models which exhibit chaotic dynamics.

John Argyris, Gunter Faust, Maria Haase, *An Exploration of Chaos, Texts on Computational Mechanics*, Vol. VII (New York: North-Holland, 1994).

Offered as introductory text on Chaos Theory, this book targets "aspiring physicists and engineers". A good deal of general theory precedes a review of physical and mechanical applications. They claim to assume no deep math background, but you really need more than a casual familiarity with differential equations and vector calculus to follow along. The book has several strengths: a detailed discussion of the logistic

map; a nice **compilation of classes of bifurcations**; an interesting analysis of bone formation and regrowth. The applications are presented in fine detail, making the results reproducible for interested readers. **Most important:** the authors outline a **general process of theoretical and numerical investigation** appropriate for technical applications of Chaos results. They conclude with a spectacular bibliography of primary technical sources.

Katz, Richard A., ed., *The Chaos Paradigm: Developments and Applications in Engineering and Science*, American Institute of Physics (AIP) Conference Proceedings 296, Mystic, CT, 1993 (New York: AIP Press, 1994).

This is a terrific survey of current research sponsored by the Office of Naval Research and the Naval Undersea Warfare Center. The list of participants is a useful Who's Who of many current research areas; the articles sample the diverse fields where DOD engages in active research. Anywhere from two to four brief articles on each of the following topics cover: Math Foundations of Chaos, Mechanical Sources of Chaos, Turbulence, Control of Chaos, Signal Modeling, Noise Reduction, Signal Processing, and Propagation Modeling.

Todor Tagarev, Michael Dolgov, David Nicholls, Randal C. Franklin and Peter Axup, *Chaos in War: Is It Present and What Does It Mean?* Report to Air Command and Staff College, Maxwell AFB, Alabama, Academic Year 1994 Research Program, June 1994.

This was the best in-depth report examining **historical data** for evidence of Chaos. The authors find chaotic dynamics in tactical, operational and strategic levels of military activity:

- aircraft loss data for the entire Vietnam War;
- Allied casualty data in their advance through western Europe in World War II;
- historical US defense spending.

The paper's **greatest strength**: the discussion of **data collection and analysis**, the **obstacles** the authors encountered, and details of their **search process**. This full report was much more meaningful than the subsequent article they distilled for the *Airpower Journal* in late 1994.¹²⁴ Both the short article and the full essay contain some substantial technical errors in the basics of Chaos, but the authors have clearly done their homework.

T. Matsumoto et al., *Bifurcations: Sights, Sounds and Mathematics* (New York: Springer-Verlag, 1993).

This textbook generally expects the reader to have an extensive mathematical background, but it starts with a fantastic section describing **simple electronic circuits** which exhibit a vast array of chaotic dynamics. This is a great reference for those with

access to or interest in electronics applications. The book also includes a thorough study of various **classes of bifurcations** common to many dynamical systems.

Edward Ott, Tim Sauer and James A. Yorke eds., *Coping with Chaos: Analysis of Chaotic Data and the Exploitation of Chaotic Systems* (New York: John Wiley & Sons, Inc., 1994).

Topicwise, this book is the best end-to-end compilation of chapters and articles, mostly published in other sources, which go from theoretical background to data analysis and applications. The text includes more recent work on practical suggestions for calculating dimensions, Lyapunov exponents, time embeddings, and control techniques. While the collection of articles is virtually all reprinted from primary sources, it's a good collection and can save an interested reader many hours of digging through periodical holdings. This book does require a solid background in vector calculus and differential equations, but is very practical. The articles are generally at the level of papers from *Physical Review* and *Physical Review Letters*. Again, you'll find a spectacular bibliography in this one.

G. Mayer-Kress, ed., *Dimensions and Entropies in Chaotic Systems: Quantification of Complex Behavior*, Proceedings of an International Workshop at the Pecos River Ranch, New Mexico, September 11-16, 1985 (New York: Springer-Verlag, 1986).

This thin text publishes the collection of papers contributed to the workshop cited. Take note: this is an older reference describing some of the early results of Chaos

calculations. However, it presents a comprehensive review of techniques, modifications and improvements, and explanations of how they are related. The papers cover the intense details of how to calculate, in both theory and experiment: fractal measures, fractal dimensions, entropies, Lyapunov exponents. This is a **highly** technical work, **not** for the casual reader or weak of heart, and not a good place to first learn about these measurements. However, it **is** necessary reading for serious analysts embarking on numerical explorations of dynamical systems.

Michael F. Barnsley and Lyman P. Hurd, *Fractal Image Compression* (Wellesley, Massachusetts: AK Peters, Ltd, 1993).

Perhaps more dense (i.e. slower) reading than Barnsley's first text, *Fractals Everywhere*, this fine book focuses appropriately on only those details required to understand the fractal compression techniques patented by Iterated Systems Inc. It's a very thorough presentation, pleasant reading, and the text includes sample C source code and many demonstrations of decompressed images.

Saul Krasner, ed., *The Ubiquity of Chaos* (Washington DC: American Association for the Advancement of Science (AAAS), 1990).

This is another nice review of Chaos applications in a wide variety of disciplines: Dynamical Systems, Biological Systems, Turbulence, Quantized Systems, Global Affairs, Economics and the Arms Race, Celestial Systems. Great bibliographies follow each

individual article; most chapters have **not** been published elsewhere, as is often the case in similar collections of contributions by many independent authors.

Naval Research Reviews, Office of Naval Research (ONR), Vol XLV (3), 1993.

This special issue is devoted to ONR-sponsored research in engineering applications of Chaos. Nice overview articles cover the following topics: Controlling Chaos, Noisy Chaos, Communicating with Chaos, Nonlinear Resonance in Neurophysiological Systems, and Image Compression.

SIX

Further Questions to Research

I've assembled, in this chapter, a broad collection of research topics that deserve more careful study. For the benefit of students and prospective research advisors, I've done my best to form the questions and issues into packages small enough to address within a short research term during in-residence Professional Military Education.

Complexity: The Next Big Step. My report discusses how simple models can display complex behavior. However, once we develop a good intuition for Chaos, other questions arise immediately. Here's a peek at one of the central issues, only slightly oversimplified. **Fact:** fluids tend to move chaotically. The very nature of their dynamics makes them extremely sensitive to small disturbances. Now, the mixture inside a chicken egg is a fluid; that mixture is surely subjected to bumps and jostles during the formation of the baby chick inside. **Question:** if the fluid is chaotic, and its motion and behavior is so unpredictable, how does the creature inside **always** come out a **chicken**?!

The answers to questions like these are the subject of the (even more recent) science of Complexity. You may consider researching complexity and self-organization. When and why do complicated systems sometimes organize themselves to behave "simply"? Which results of **this** theory are relevant for military decision makers?

Exponents. Identify a few specific military systems, perhaps within the context of a war game or through historical data, and calculate some Lyapunov exponents to compare the systems' relative sensitivity to perturbation. Prioritize the importance of various systems for protection or attack.

Additional Dynamics. In Robert Axelrod's aggregation model, he successfully predicts the end states of two multi-party alliances, but there's still room to consider the **dynamics** of these alliances. How long do the alignments take to form? How stable are the end states? What sort of perturbations break the alliances? The analysis is static only, so far, though he does discuss the presence of "basins of attraction" of the end-state configurations.¹²⁵

Feedback. Where **are** the feedback loops in current and future military systems? Consider both friendly and hostile systems. Also investigate both mechanical and social systems. Examine the strategic options for imposing feedback on these systems, and protecting the systems from unwanted feedback. What behaviors and system transitions should we expect?

Sensors. What sort of sensors can we identify as vulnerable to imposed feedback? Where are they and how do they operate? What creative strategies can we devise to exploit or reduce their sensitivity to disturbances?

War games. Can we replace random variables in war games with simple chaotic equations that produce comparable distributions? Can the underlying equations lead to clues about which parameters are most important? How do our games behave now? Can any be driven into Chaos with the right combination of parameters? For a detailed discussion of the use of historical data for battlefield predictions, see Col T. N. Dupuy's book, *Numbers, Predictions & War*.¹²⁶ He thoroughly discusses the issues of data compilation, modeling, prediction, and he tabulates exhaustive lists of relevant battlefield parameters.

The Nonlinear Battlefield. Maj Sean B. MacFarland, at the Army School of Advanced Military Studies (SAMS), defines "operational non-linearity" as the dispersed state of a combat force characterized by a complex of interconnecting fire positions and carefully sighted long-range weapons.¹²⁷ His paper highlights the difference between **geometric nonlinearity** and systemic (dynamical) nonlinearity. If we think of a force's physical disposition as its "state" in a combat system, old ideas of Forward Edge of the Battle Area may be replaced by emerging perspectives of overlapping attractors.

Maj J. Marc LeGare, also at SAMS, proposed operations on the nonlinear battlefield organized in a "Tactical Cycle": disperse, mass, fight, redisperse, and reconstitute.¹²⁸ Could we structure this cycle to protect our own dynamics, and take advantage of enemy cycles to break down their systems? If our forces are limited, can we exploit these cycles to apply our force efficiently and control the combat dynamics? What kind of small perturbations could we impose on such a combat system? The

answers to some of these questions may spring from other articles that consider the tactics of potential adversaries on the nonlinear battlefield.¹²⁹

We should also note that the idea of dispersed, nonsequential operations is not new. In 1967, Rear Admiral J.C. Wylie, contrasted two very different kinds of strategies. One is **sequential**, a series of visible discrete steps that follow one another deliberately through time. The other is **cumulative**, “the less perceptible minute accumulation of little items piling one on top of the other until at some unknown point the mass of accumulated actions may be large enough to be critical.” He observes that, in the Pacific from 1941 to 1945, “**we were not able to predict the compounding effect** of the cumulative strategy (individual submarine attacks on Japanese tonnage) as it operated concurrently with and was enhanced by the sequential strategy [of the drive up the Pacific islands].”¹³⁰ Strategies like these may lend themselves to deeper analysis through Chaos Theory.

Case Studies.

*For want of a nail the shoe is lost,
For want of a shoe the horse is lost,
For want of a horse the rider is lost,
For want of a rider the battle is lost,
For want of the battle the war is lost,
For want of the war the nation is lost,
All for the want of a horseshoe nail.*

George Herbert (1593-1632)

We already noted one effort to examine the Mayaguez crisis in the light of Chaos results. This was, of course, only a rough beginning. Several historical case studies (all entitled *For Want of a Nail!*) highlight the sensitivity of combat events to small “disturbances.” The following references provide a list of candidate cases to consider for further Chaos analyses.

Robert Sobel composes a detailed counterfactual book of what would have happened had Burgoyne **held** Saratoga in the American Revolution.¹³¹ Hugh R. Wilson studies the ineffective application of economic sanctions against Italy, in the winter of 1935-36, during the Italian military excursion into Ethiopia.¹³² Hawthorne Daniel, investigates the influence of logistics on war in several interesting case studies:¹³³

- American Revolution: New Jersey 1776; Lake Champlain and the Hudson River 1777
- Peninsular War: Spain and Portugal 1808 to 1814
- The Moscow Campaign: Russia 1812
- American Civil War: 1861 to 1865
- Sudan Campaign: The Upper Nile 1896 to 1898
- The Allied Invasion of Occupied Europe, WWII: 1944 and 1945

Bibliography. With the recent explosion in Chaos resources, the preparation of a comprehensive bibliography would provide a tremendous service to the general research community. The reference lists in the texts I noted above are a great place to start. Many book reviews are also available to guide examinations of the most recent texts.

Write More! Above all, you should regard this essay as one voice in a continuing conversation. It will always be valuable for you to document other interesting thoughts and research. Please continue the conversation. In particular, there's plenty of room for open debate on any issues you feel I've missed or overstated. It would also be a great help to publish additional military applications of which you may be aware. I look forward to reading **your** thoughts.

SEVEN

Conclusion

This report has focused on those issues of Chaos Theory essential to military decision makers. The new science of Chaos studies behavior that is characterized by erratic fluctuations, sensitivity to disturbances, and long-term unpredictability. This paper conducted a thorough review of Chaos applications in military affairs, and hopefully, corrected some deficiencies in current publications on Chaos.

We centered our Chaos study in three areas. First, we reviewed the fundamentals of chaotic dynamics, to build some intuition for Chaos. Second, we surveyed the current military technologies that are prone to chaotic dynamics. Third, we saw how the universal properties of chaotic systems point to practical suggestions for applying Chaos results to strategic thinking and decision making. The power of Chaos comes from this universality: not just the vast number of chaotic systems, but the common types of behaviors and transitions that appear in completely unrelated systems. As a result, recent recognition of Chaos in social systems offers new opportunities to apply these results to problems in decision making, strategic planning and policy formulation.

The evidence is clear: chaotic dynamics pervade the dynamics of military affairs. The implications of Chaos Theory offer an extraordinary range of options unavailable only 20 years ago. Not only do current military systems naturally exhibit chaotic

dynamics, but many systems are vulnerable to new strategies that exploit Chaos results. Because of the theory's tremendous potential, every military leader needs to be familiar with the fundamentals of Chaos in order to recognize Chaos when it occurs, expect chaotic dynamics in military systems, and exploit the vast array of tools for diagnosing and controlling those dynamics.

Appendix

What does it mean to be *Random*?

Our usual connotations of randomness carry images of erratic, completely unpredictable behavior. For a fair die on a craps table, **randomness** means that **sooner or later**, that die will roll to a 6. It means there is no chance of that die rolling a string of 1's forever. If that were the case, the die would be very predictable, and thus, not random.

To be more precise, I'll borrow an explanation by Robert Batterman in his article, "Defining Chaos."¹³⁴ Let's start with an infinite string of perfectly alternating digits:

0 1 0 1 0 1 0 1

How much information does it take to recognize, transmit or repeat this string? Suppose we only had access to a brief list of the first few elements of the sequence. Could we draw any conclusions about the system's behavior?

0 nope

0 1 nope

0 1 0	hmmm, we begin to see a pattern
0 1 0 1	looks a little regular, but can't tell yet
0 1 0 1 0	we can start to guess some regularity. . . .

After 20, or 50, or 1000 new pieces of information (additional digits in the observed string) we think we have it: this string of data has period two; we only need three pieces of information to repeat the string:

1. Print 0.
2. Print 1.
3. Repeat steps 1 and 2.

Sure: lather, rinse, repeat. If we follow these steps, we're confident we can completely replicate the series. Now, if we don't know where or how the series was generated, we can not be positive of its perfect periodicity. Nonetheless, as we get more and more information, our confidence in our analysis improves.

So how would we characterize a *random* string of data? In terms of our data string, it means we would need the ENTIRE infinite string—that is, an **infinite list of instructions**—in order to accurately reproduce the original infinite data set. This requirement for an unending set of instructions, to communicate or reproduce the data, is sometimes offered as a formal definition for *randomness*.

Endnotes

- ¹ Michael F. Barnsley, *Fractals Everywhere*, 1st ed. (San Diego: Academic Press, 1988), 1.
- ² James Gleick, *Chaos: Making a New Science* (New York: Viking Penguin, 1987), 304.
- ³ Tagarev et al., "Chaos in War: Is It Present and What Does It Mean?" Report to Air Command and Staff College, Maxwell Air Force Base, Alabama, Academic Year 1994 Research Program, June 1994, 52.
- ⁴ Gleick, 69.
- ⁵ Department of Defense Critical Technologies Plan, for the Committees on Armed Services, U.S. Congress, 1 May 1991 [DTIC AD-A234 900], p I-3.
- ⁶ David F. Andersen, "Foreword: Chaos in System Dynamics Models," *System Dynamics Review* 4 (1-2), 1988, 6-7.
- ⁷ Gleick, 187.
- ⁸ Gleick, 43.
- ⁹ J.M.T. Thompson and H.B. Stewart, *Nonlinear Dynamics and Chaos* (New York: John Wiley & Sons, 1986), 291-310.
- ¹⁰ Gleick, 42.
- ¹¹ "NOVA: The Strange New Science of Chaos," PBS television broadcast NOVA #1603, 31 January 1989, produced by WGBH, Boston, Massachusetts.
- ¹² Gleick, 262-267.
- ¹³ John Argyris, et al., *An Exploration of Chaos*, Texts on Computational Mechanics, Vol. VII (New York: North-Holland, 1994) ix.
- ¹⁴ Gleick, 266.
- ¹⁵ Argyris et al., 65.
- ¹⁶ Gleick, 60-61.
- ¹⁷ Heinz-Otto Peitgen et al., *Chaos and Fractals: New Frontiers of Science* (New York: Springer-Verlag, 1992), 697.
- ¹⁸ Ibid., 698.
- ¹⁹ Ibid., 514.
- ²⁰ Gleick, 24.
- ²¹ Barnsley, *Fractals Everywhere*, 1st ed., 103.
- ²² J.A. Dewar et al., "Non-Monotonicity, Chaos, and Combat Models," RAND Library Collection (Santa Monica, California: RAND, 1991).

- ²³ Barnsley, *Fractals Everywhere* 1st ed., 281.
- ²⁴ T. Matsumoto, *Bifurcations: Sights, Sounds and Mathematics* (New York: Springer-Verlag, 1993).
- ²⁵ Peitgen et al., 59.
- ²⁶ Gleick, 48.
- ²⁷ Gottfried Mayer-Kress, "Chaos and Crises in International Systems," Lecture at SHAPE Technology Symposium on Crisis Management, Mons, Belgium: 19-20 March 1992.
- ²⁸ Paul E. Rapp, et al., "Dynamical Characterization of Brain Electrical Activity," in Saul Krasner ed., *The Ubiquity of Chaos*, (Washington, D.C.: American Association for the Advancement of Science (AAAS), 1990), 10.
- ²⁹ Peitgen et al., 526.
- ³⁰ Tagarev et al., 51-53.
- ³¹ Campbell and Mayer-Kress, 84.
- ³² Gleick, 119.
- ³³ Thompson, 291-310.
- ³⁴ Solomon C.S. Yim, "Chaotic and Random Responses of Ocean Structural Systems," Office of Naval Research Young Investigator Award, Research Project N00014-88-K-0729, 1 March 1992 [DTIC AD-A247 561].
- ³⁵ Zeki Okan Oral, "Hopf Bifurcations in Path Control of Marine Vehicles," Thesis, Naval Postgraduate School, Monterey, California: June 1993 [DTIC AD-A268 818].
- ³⁶ Michael F. Barnsley and Lyman P. Hurd, *Fractal Image Compression* (Wellesley, Massachusetts: AK Peters, Ltd, 1993).
- ³⁷ Yuval Fisher et al., "Fractal Image Compression," Quarterly Progress Report 8-12-91 (San Diego, California: NETROLOGIC, Inc., July 1991) [DTIC AD-A240 636].
- ³⁸ B.H. Tongue and K. Gu, "A Higher Order Method of Interpolated Cell Mapping," *Journal of Sound and Vibration* **125** (169), 1988.
- ³⁹ Max-Olivier Hongler, *Chaotic and Stochastic Behaviour in Automatic Production Lines* (Berlin: Springer-Verlag, 1994).
- ⁴⁰ Department of Defense Critical Technologies Plan, May 1991.
- ⁴¹ Kurt Wiesenfeld et al., "Observation of Antiphase States in a Multimode Laser," *Physical Review Letters* **65** (14), 1 October 1990, 1749-1752.
- ⁴² Tagarev et al., 32.
- ⁴³ Frank Heppner and Ulf Grenander, "A Stochastic Nonlinear Model for Coordinated Bird Flocks," in Saul Krasner, ed., *The Ubiquity of Chaos* (Washington, D.C.: American Association for the Advancement of Science (AAAS), 1990), 233-238.
- ⁴⁴ Grossman and Mayer-Kress, 701-704.
- ⁴⁵ Ibid.
- ⁴⁶ Wheatley, 7.
- ⁴⁷ Headquarters Department of the Army, *Operations*, FM100-5 (Washington, D.C.: 14 June 1993), p. 2-6.

- ⁴⁸ Department of the Navy, *Warfighting*, FMFM-1 (Washington, D.C.: 6 March 1989), 66-67.
- ⁴⁹ Prof Michael I. Handel, "German Strategic Alternatives," Lecture at U.S. Naval War College, Newport, R.I.: 26 September 1994.
- ⁵⁰ Prof Edward Teller, comments at The Tenth Annual Space Symposium, Broadmoor Hotel, Colorado Springs, Colorado: February 1994.
- ⁵¹ J.P. Crutchfield, "Knowledge and Meaning: Chaos and Complexity," in Lui Lam and Vladimir Naroditsky, eds., *Modeling Complex Phenomena* (New York: Springer-Verlag, 1992), 68.
- ⁵² Andersen, 3-13.
- ⁵³ Interview with Dr Stephen O. Fought, National Security Decision Making Department, Naval War College, Newport, R.I.: 1 February 1995.
- ⁵⁴ Andreas S. Weigend and Neil A. Gerschenfeld, eds., *Time Series Prediction: Forecasting the Future and Understanding the Past*, Proceedings of the NATO Advanced Research Workshop on Comparative Time Series Analysis held in Santa Fe, New Mexico, May 14-17, 1992, Proceedings Vol. XV, Santa Fe Institute Studies in the Sciences of Complexity (Reading, Massachusetts: Addison-Wesley, 1994), 175-193.
- ⁵⁵ Weigend, 191.
- ⁵⁶ Ibid.
- ⁵⁷ M. Casdagli, "Nonlinear Forecasting, Chaos and Statistics," in Lui Lam and Vladimir Naroditsky, eds., *Modeling Complex Phenomena* (New York: Springer-Verlag, 1992), 131-152.
- ⁵⁸ Edward Ott et al., eds., *Coping with Chaos: Analysis of Chaotic Data and the Exploitation of Chaotic Systems* (New York: John Wiley & Sons, 1994).
- ⁵⁹ J.D. Farmer and John J. Sidorowich, "Exploiting Chaos to Predict the Future and Reduce Noise," in Y.C. Lee, ed., *Evolution, Learning and Cognition* (Teaneck, New Jersey: World Scientific Publishing, 1988), 277-330.
- ⁶⁰ William W. Taylor, "Chaotic Evolution and Nonlinear Prediction in Signal Separation Applications," RAND Report P-7769 (Santa Monica, California: RAND, April 1994).
- ⁶¹ Interview with Dr John Hanley, Program Director of U.S. Naval War College Strategic Studies Group, and CDR Bill Millward, USN, Naval War College, Newport, RI: 16 December 1994.
- ⁶² Thompson, xii.
- ⁶³ Major Patrick Kelly, 38.
- ⁶⁴ Edward Ott et al., "Controlling Chaos," *Physical Review Letters* **64** (11), 12 March 1990, 1196-1199.
- ⁶⁵ Ibid.
- ⁶⁶ Troy Shinbrot et al., "Using Small Perturbations to Control Chaos," *Nature* **363**, 3 June 1993, 415.
- ⁶⁷ Ott et al., *Coping with Chaos*.
- ⁶⁸ Elizabeth Bradley, "Control Algorithms for Chaotic Systems," MIT Artificial Intelligence Laboratory, AI Memo No. 1278 (Cambridge, Massachusetts: MIT, March 1991) [DTIC AD-A260 067].
- ⁶⁹ Shinbrot et al., 415.

- ⁷⁰ W.L. Ditto et al., "Experimental Control of Chaos," *Physical Review Letters* **65** (26), 24 December 1990, 3211-3214.
- ⁷¹ Henry D.I. Abarbanel, "Nonlinearity and Chaos at Work," *Nature* **364**, 19 August 1993, 672-673.
- ⁷² Martinus M. Sarigul-Klijn, "Application of Chaos Methods to Helicopter Vibration Reduction Using Harmonic Control" Ph.D. Dissertation, Naval Postgraduate School, Monterey, California, March 1990 [DTIC AD-A226 736].
- ⁷³ Ed Paisley, "Out of Chaos, Profits," *Far Eastern Economic Review*, 7 October 1993, 84.
- ⁷⁴ Raj Roy et al., "Dynamical Control of a Chaotic Laser: Experimental Stabilization of a Globally Coupled System," *Physical Review Letters* **68**, 2 March 1992, 1259-1262.
- ⁷⁵ Jeffrey J. Leader, "Chaotic Keystream Generators for Additive Stream Ciphers," Technical Report NPS-MA-93-015 (Monterey, California: Naval Postgraduate School, Jan 1993-April 1993) [DTIC AD-A265 824].
- ⁷⁶ Joseph Neff and Thomas L. Carroll, "Circuits That Get Chaos In Sync," *Scientific American*, August 1993, 120-122.
- ⁷⁷ Bradley, "Taming Chaotic Circuits."
- ⁷⁸ Ott et al., "Controlling Chaos."
- ⁷⁹ Lt Col (retired) Price T. Bingham, USAF, "The Air Force's New Doctrine," *Military Review* (Fort Leavenworth, KS: US Army Command and General Staff College, November 1992), 13-14.
- ⁸⁰ Gleick, 278.
- ⁸¹ Thompson, xi.
- ⁸² Wheatley, 126.
- ⁸³ Jim Yorke, "Calculated Trajectories in Chaotic Systems," Lecture at Georgia Institute of Technology, School of Mathematics, Atlanta, Georgia: Fall 1989.
- ⁸⁴ Argyris et al., 232.
- ⁸⁵ Ibid., 72.
- ⁸⁶ Ralph Abraham et al., "Computational Unfolding of Double-Cusp Models of Opinion Formation," *International Journal of Bifurcation and Chaos* **1** (2), 1991, 417-430.
- ⁸⁷ Y.Y. Azmy and V. Protopopescu, "Two Dimensional Maps Generated by Competitive Systems," Oak Ridge National Laboratory (ORNL) TM-11026 (Oakridge, Tennessee: ORNL, February 1989).
- ⁸⁸ JA. Dewar et al., "Non-Monotonicity, Chaos, and Combat Models," RAND Library Collection (Santa Monica, California: RAND, 1991).
- ⁸⁹ Dewar, 42.
- ⁹⁰ John T. Dockery and A.E.R. Woodcock, *The Military Landscape* (Cambridge, England: Woodhead Publishing, 1993).
- ⁹¹ Dockery and Woodcock, 138.
- ⁹² Ibid., 137.
- ⁹³ Glenn E. James et al., "Elimination of Chaos in an Intracavity-Doubled Nd:YAG Laser," *Optics Letters* **15** (20), 15 October 1990, 1141-1143.

- ⁹⁴ Alvin M. Saperstein, "Chaos and the Making of International Security Policy," in Saul Krasner, ed., *The Ubiquity of Chaos* (Washington, D.C.: American Association for the Advancement of Science (AAAS), 1990), 167-180.
- ⁹⁵ Maj David Nicholls, USAF, and Maj Todor D. Tagarev, Bulgarian Air Force, "What Does Chaos Theory Mean for Warfare?" *Airpower Journal*, Fall 1994, 48-57.
- ⁹⁶ Capt James R. Stright, USAF, "Embedded Chaotic Time Series: Applications in Prediction and Spatio-Temporal Classification," Ph.D. Dissertation, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio, June 1994 [DTIC AD-A280 690].
- ⁹⁷ "Project 2025 Briefing for the Secretary of Defense and the Service Chiefs," Institute for National Strategic Studies, National Defense University, 1994.
- ⁹⁸ Saperstein, 167-180.
- ⁹⁹ Gleick, 108.
- ¹⁰⁰ Telephone conversations with Lt Col Gerald Diaz, USAF, Defense Airborne Reconnaissance Office, Washington D.C.: November 1994.
- ¹⁰¹ Patrick Seitz, "C-Band Shortage Forces Scramble for Alternatives," *Space News* 5 (43), 7-13 November 1994, 1.
- ¹⁰² Michael F. Barnsley, *Fractals Everywhere*, 2d ed. (Cambridge, Massachusetts: Academic Press Professional, 1993), 94-95.
- ¹⁰³ Telephone interview with Rick Darby, Iterated Systems, Inc., Norcross, Georgia: 28 November 1994.
- ¹⁰⁴ Information based on published descriptions of off-the-shelf products available from Iterated Systems, Inc., 5550-A Peachtree Parkway, Suite 650, Norcross, GA 30092.
- ¹⁰⁵ Gleick, 262.
- ¹⁰⁶ Lt Col Theodore H. Mueller, USA, "Chaos Theory: The Mayaguez Crisis," U.S. Army War College, Military Studies Program, Carlisle Barracks, Pennsylvania, 15 March 1990.
- ¹⁰⁷ Joshua M. Epstein, "On the Mathematical Biology of Arms Races, Wars, and Revolutions," in Lynn Nadel and Daniel L. Stein, eds., "1992 Lectures in Complex Systems," Lecture Vol V, Sante Fe Institute Studies in the Sciences of Complexity (Reading, Massachusetts: Addison-Wesley, 1993), 425-436.
- ¹⁰⁸ John D. Sterman, "Deterministic Chaos in Models of Human Behavior: Methodological Issues and Experimental Results," *System Dynamics Review* 4 (1-2), 1988, 148-178.
- ¹⁰⁹ Gregoire Nicolis and Ilya Prigogine, *Exploring Complexity* (New York: W.H. Freeman and Company, 1989), 232.
- ¹¹⁰ Abraham et al., 417-430.
- ¹¹¹ Robert Axelrod and D. Scott Bennett, "A Landscape Theory of Aggression," *British Journal of Political Science*, Part 2, 23, April 1993, 211-233.
- ¹¹² Diana Richards, "Is Strategic Decision Making Chaotic?" *Behavior Science* 35, 1990, 219-232.
- ¹¹³ Todor Tagarev, et al.
- ¹¹⁴ Sterman, 148-178.
- ¹¹⁵ Ibid., 155.

- ¹¹⁶ William J. Ray, "EEG and Chaos: Description of Underlying Dynamics and Its Relation to Dissociative States," Final Progress Report: NASA Grant #NAG-1-1441, February 1994 [NASA-CR-195171].
- ¹¹⁷ Hal Gregersen and Lee Sailer, "Chaos Theory and Its Implications for Social Science Research," *Human Relations* **46** (7), 1993, 777-802.
- ¹¹⁸ Todor Tagarev et al..
- ¹¹⁹ Sterman, 172.
- ¹²⁰ Grossman and Mayer-Kress, 701-704.
- ¹²¹ Dick Axelrod, "Getting Everyone Involved: How One Organization Involved Its Employees, Supervisors and Managers in Redesigning the Organization," *The Journal of Applied Behavioral Science* **28** (4), December 1992, 499-509.
- ¹²² Sterman, 148-178.
- ¹²³ David F. Andersen and Jeppe Sturis, "Chaotic Structures in Generic Management Models: Pedagogical Principles and Examples," *System Dynamics Review* **4** (1-2), 1988, 218-245.
- ¹²⁴ Nicholls and Tagarev.
- ¹²⁵ Robert Axelrod and Bennett.
- ¹²⁶ Col T. N. Dupuy, *Numbers, Predictions & War: The Use of History to Evaluate and Predict the Outcome of Armed Conflict*, 2d ed. (Fairfax, VA: HERO Books, 1985).
- ¹²⁷ Maj Sean B. MacFarland, USA, "Non-Linear Operations: A New Doctrine for a New Era," School of Advanced Military Studies, US Army Command and General Staff College, Fort Leavenworth, Kansas, Second Term Academic Year 1993-94, 20 April 1994 [DTIC AD-A284 137], 15.
- ¹²⁸ Maj J. Marc LeGare, USA, "Paradigm Found—The Nuclear and Nonlinear Battlefields," School of Advanced Military Studies, US Army Command and General Staff College, Fort Leavenworth, Kansas, First Term Academic Year 1992-93, February 1993 [DTIC AD-A264 451], 23.
- ¹²⁹ LTC Lester W. Grau, USA, "Soviet Non-Linear Combat: The Challenge of the 90s," Soviet Army Studies Office, US Army Combined Arms Center, Fort Leavenworth, Kansas, September 1990.
- ¹³⁰ Rear Admiral J.C. Wylie, USN, "Military Strategy: A General Theory of Power Control," in George Edward Thibault, ed., *The Art and Practice of Military Strategy* (New York: Rutgers, The State University, 1967), 200-201.
- ¹³¹ Robert Sobel, *For Want of a Nail: If Burgoyne Had Won at Saratoga* (New York: MacMillan Co., 1973).
- ¹³² Hugh R. Wilson, Jr., *For Want of a Nail: The Failure of the League of Nations in Ethiopia* (New York: Vantage Press, 1959).
- ¹³³ Hawthorne Daniel, *For Want of a Nail: The Influence of Logistics on War* (New York: McGraw-Hill, 1948).
- ¹³⁴ Robert W. Batterman, "Defining Chaos," *Philosophy of Science* **60** (1993) 56.

Bibliography

Books, Edited Collections and Proceedings

- Argyris, John, Gunter Faust and Maria Haase. *An Exploration of Chaos*. Texts on Computational Mechanics, Vol. VII. New York: North-Holland, 1994.
- Barnsley, Michael F. *Fractals Everywhere*, 1st ed. San Diego: Academic Press, 1988.
- Barnsley, Michael F. *Fractals Everywhere*, 2d ed. Cambridge, Massachusetts: Academic Press Professional, 1993.
- Barnsley, Michael F. and Lyman P. Hurd. *Fractal Image Compression*. Wellesley, Massachusetts: AK Peters, Ltd, 1993.
- Casdagli, M. "Nonlinear Forecasting, Chaos and Statistics," in Lui Lam and Vladimir Naroditsky, eds., *Modeling Complex Phenomena*. New York: Springer-Verlag, 1992, 131-152.
- Crutchfield, J.P. "Knowledge and Meaning: Chaos and Complexity," in Lui Lam and Vladimir Naroditsky, eds., *Modeling Complex Phenomena*. New York: Springer-Verlag, 1992, 66-101.
- Daniel, Hawthorne. *For Want of a Nail: The Influence of Logistics on War*. New York: McGraw-Hill, 1948.
- Dockery, John T. and A.E.R. Woodcock. *The Military Landscape*. Cambridge, England: Woodhead Publishing, 1993.
- Dupuy, Col T. N. *Numbers, Predictions & War: The Use of History to Evaluate and Predict the Outcome of Armed Conflict*, 2 ed. Fairfax, VA: HERO Books, 1985.
- Epstein, Joshua M. "On the Mathematical Biology of Arms Races, Wars, and Revolutions," in Lynn Nadel and Daniel L. Stein, eds., "1992 Lectures in Complex Systems." Lecture Vol V, Sante Fe Institute Studies in the Sciences of Complexity. Reading, Massachusetts: Addison-Wesley, 1993, 425-436.
- Farmer, J.D., and John J. Sidorowich. "Exploiting Chaos to Predict the Future and Reduce Noise," in Y.C. Lee, ed., *Evolution, Learning and Cognition*. Teaneck, New Jersey: World Scientific Publishing, 1988, 277-330.
- Gleick, James. *Chaos: Making a New Science*. New York: Viking Penguin Inc, 1987.
- Heppner, Frank and Ulf Grenander. "A Stochastic Nonlinear Model for Coordinated Bird Flocks," in Saul Krasner, ed., *The Ubiquity of Chaos*. Washington, D.C.: American Association for the Advancement of Science (AAAS), 1990, 233-238.

- Hongler, Max-Olivier. *Chaotic and Stochastic Behaviour in Automatic Production Lines*. Berlin: Springer-Verlag, 1994.
- Katz, Richard A., ed. *The Chaos Paradigm: Developments and Applications in Engineering and Science*. American Institute of Physics (AIP) Conference Proceedings 296, Mystic, CT, 1993. New York: AIP Press, 1994.
- Nicolis, Gregoire, and Ilya Prigogine. *Exploring Complexity*. New York: W.H. Freeman and Company, 1989.
- Mayer-Kress, G., ed. *Dimensions and Entropies in Chaotic Systems: Quantification of Complex Behavior*. Proceedings of an International Workshop at the Pecos River Ranch, New Mexico, September 11-16, 1985. New York: Springer-Verlag, 1986.
- Matsumoto, T. *Bifurcations: Sights, Sounds and Mathematics*. New York: Springer-Verlag, 1993.
- Ott, Edward, Tim Sauer and James A. Yorke, eds. *Coping with Chaos: Analysis of Chaotic Data and the Exploitation of Chaotic Systems*. New York: John Wiley & Sons, 1994.
- Peitgen, Heinz-Otto, Hartmut Jürgens and Dietmar Saupe. *Chaos and Fractals: New Frontiers of Science*. New York: Springer-Verlag, 1992.
- Rapp, Paul E., Theodore R. Bashore, Irwin D. Zimmerman, Jacques M. Martinerie, Alfonso M. Albano and Alistair I. Mees. "Dynamical Characterization of Brain Electrical Activity," in Saul Krasner, ed., *The Ubiquity of Chaos*. Washington, D.C.: American Association for the Advancement of Science (AAAS), 1990, 10-22.
- Saperstein, Alvin M. "Chaos and the Making of International Security Policy," in Saul Krasner, ed., *The Ubiquity of Chaos*. Washington, D.C.: American Association for the Advancement of Science (AAAS), 1990, 167-180.
- Sauer, Tim. "Time Series Prediction by Using Delay Coordinate Embedding," in Andreas S. Weigend and Neil A. Gerschenfeld, eds., *Time Series Prediction: Forecasting the Future and Understanding the Past*. Proceedings of the NATO Advanced Research Workshop on Comparative Time Series Analysis held in Santa Fe, New Mexico, May 14-17, 1992, Proceedings Vol. XV, Santa Fe Institute Studies in the Sciences of Complexity. Reading, Massachusetts: Addison-Wesley, 1994.
- Sobel, Robert. *For Want of a Nail...: If Burgoyne Had Won at Saratoga*. New York: MacMillan Co., 1973.
- Taylor, William W. "Chaotic Evolution and Nonlinear Prediction in Signal Separation Applications." RAND Report P-7769. Santa Monica California: RAND, April 1994.
- Thompson, J.M.T. and H.B. Stewart. *Nonlinear Dynamics and Chaos*. New York: John Wiley & Sons, 1986.
- Weigend, Andreas S., and Neil A. Gerschenfeld, eds. *Time Series Prediction: Forecasting the Future and Understanding the Past*. Proceedings of the NATO Advanced Research Workshop on Comparative Time Series Analysis held in Santa Fe, New Mexico, May 14-17, 1992. Proceedings Vol. XV, Santa Fe Institute Studies in the Sciences of Complexity. Reading, Massachusetts: Addison-Wesley, 1994.

Wheatley, Margaret. *Leadership and the New Science*. San Francisco: Berrett-Koehler Publishers, 1992.

Wilson, Hugh R., Jr. *For Want of a Nail: The Failure of the League of Nations in Ethiopia*. New York: Vantage Press, 1959.

Wylie, Rear Admiral J.C., USN. "Military Strategy: A General Theory of Power Control," in George Edward Thibault, ed., *The Art and Practice of Military Strategy*. New York: Rutgers, The State University, 1967, 196-203.

Periodicals

Abarbanel, Henry D.I. "Nonlinearity and Chaos at Work." *Nature* **364**, 19 Aug 1993, 672-673.

Abraham, Ralph, Alexander Keith, Matthew Koebe and Gottfried Mayer-Kress. "Computational Unfolding of Double-Cusp Models of Opinion Formation." *International Journal of Bifurcation and Chaos* **1** (2), 1991, 417-430.

Andersen, David F. "Foreword: Chaos in System Dynamics Models." *System Dynamics Review* **4** (1-2), 1988, 3-13.

Andersen, David F., and Jeppe Sturis. "Chaotic Structures in Generic Management Models: Pedagogical Principles and Examples." *System Dynamics Review* **4** (1-2), 1988, 218-245.

Axelrod, Dick. "Getting Everyone Involved: How One Organization Involved Its Employees, Supervisors and Managers in Redesigning the Organization." *The Journal of Applied Behavioral Science* **28** (4), December 1992, 499-509.

Axelrod, Robert and D. Scott Bennett. "A Landscape Theory of Aggression." *British Journal of Political Science*, Part 2, **23**, April 1993, 211-233.

Batterman, Robert W. "Defining Chaos." *Philosophy of Science* **60**, 1993, 43-66.

Bingham, Lt Col (retired) Price T., USAF. "The Air Force's New Doctrine." *Military Review*. Fort Leavenworth, KS: U.S. Army Command and General Staff College, November 1992, 13-19.

Ditto, W.L., S.N. Rauseo and M.L. Spano. "Experimental Control of Chaos." *Physical Review Letters* **65** (26), 24 December 1990, 3211-3214.

Gregersen, Hal, and Lee Sailer. "Chaos Theory and Its Implications for Social Science Research." *Human Relations* **46** (7), 1993, 777-802.

Grossman, Siegfried and Gottfried Mayer-Kress. "Chaos in the International Arms Race." *Nature* **337**, 23 February 1989, 701-704.

James, Glenn E., Evans M. Harrell II, Christopher Bracikowski, Kurt Wiesenfeld and Rajarshi Roy. "Elimination of Chaos in an Intracavity-Doubled Nd:YAG Laser." *Optics Letters* **15** (20), 15 October 1990, 1141-1143.

Mann, Steven R. "Chaos Theory and Strategic Thought." *Parameters: Journal of the U.S. Army War College*, Autumn 1992, 54-68.

Naval Research Reviews, Office of Naval Research (ONR), Vol XLV (3), 1993. Special issue devoted to Chaos applications.

Neff, Joseph and Thomas L. Carroll. "Circuits That Get Chaos In Sync." *Scientific American*, August 1993, 120-122.

Nicholls, Maj David, USAF, and Maj Todor D. Tagarev, Bulgarian Air Force. "What Does Chaos Theory Mean for Warfare?" *Airpower Journal*, Fall 1994, 48-57.

Ott, Edward, Celso Grebogi and James A. Yorke. "Controlling Chaos." *Physical Review Letters* **64** (11), 12 March 1990, 1196-1199.

Paisley, Ed. "Out of Chaos, Profits." *Far Eastern Economic Review*, 7 October 1993, 84.

Petrov, Valery, Vilmos Gaspar, Jonathan Masere and Kenneth Showalter. "Controlling Chaos in the Belousov-Zhabotinsky Reaction." *Nature* **361**, 21 January 1993, 240-243.

Richards, Diana. "Is Strategic Decision Making Chaotic?" *Behavior Science* **35**, 1990, 219-232.

Roy, Raj, T.W. Murphy, T.D. Maier, Z. Gills and E.R. Hunt. "Dynamical Control of a Chaotic Laser: Experimental Stabilization of a Globally Coupled System." *Physical Review Letters* **68**, (1992), 1259-1262.

Seitz, Patrick. "C-Band Shortage Forces Scramble for Alternatives." *Space News* **5** (43), 7-13 November 1994, 1.

Shinbrot, Troy, Celso Grebogi, Edward Ott and James A. Yorke. "Using Small Perturbations to Control Chaos." *Nature* **363**, 3 June 1993, 415.

Sterman, John D. "Deterministic Chaos in Models of Human Behavior: Methodological Issues and Experimental Results." *System Dynamics Review* **4** (1-2), 1988, 148-178.

Stringer, John. "Application of Chaos Theory to Corrosion Control." *EPRI Journal* (July/August 1993), 36-38.

Wiesenfeld, Kurt, Christopher Bracikowski, Glenn James and Rajarshi Roy. "Observation of Antiphase States in a Multimode Laser." *Physical Review Letters* **65** (14), 1 October 1990, 1749-1752.

Reports and Documents

Azmy, Y.Y. and V. Protopopescu. "Two Dimensional Maps Generated by Competitive Systems." Oak Ridge National Laboratory (ORNL) TM-11026. Oakridge, Tennessee: ORNL, February 1989.

Bradley, Elizabeth. "Control Algorithms for Chaotic Systems." MIT Artificial Intelligence Laboratory, AI Memo No. 1278. Cambridge, Massachusetts: MIT, March 1991 [DTIC AD-A260 067].

Bradley, Elizabeth. "Taming Chaotic Circuits." Massachusetts Institute of Technology (MIT) Artificial Intelligence Laboratory, AI-TR 1388. Cambridge, Massachusetts: MIT, September 1992 [DTIC AD-A259 495].

Campbell, David K. and Gottfried Mayer-Kress. "Nonlinearity and Chaos: The Challenges and Limitations of Mathematical Modeling of Environmental and Socio-Political Issues," in Proceedings of the Conference on Technology-Based Confidence Building: Energy and Environment, Center for National Security Studies (CNSS) Papers, No. 22. Los Alamos National Laboratory, New Mexico, November 1989 [LA-11728-C] 71-91.

Department of Defense Critical Technologies Plan, for the Committees on Armed Services. U.S. Congress, 1 May 1991 [DTIC AD-A234 900].

Department of the Navy. *Warfighting*, FMFM-1. Washington, D.C., 6 March 1989.

Dewar, J.A., J.J. Gillogly and M.L. Juncosa. "Non-Monotonicity, Chaos, and Combat Models." RAND Library Collection. Santa Monica, CA : RAND, 1991.

Fisher, Yuval, Ben Bielefeld, Albert Lawrence and Dan Greenwood. "Fractal Image Compression." Quarterly Progress Report 8-12-91. San Diego, California: NETROLOGIC, Inc., July 1991 [DTIC AD-A240 636].

Grau, LTC Lester W., USA. "Soviet Non-Linear Combat: The Challenge of the 90s." Soviet Army Studies Office, US Army Combined Arms Center, Fort Leavenworth, Kansas. September 1990.

Headquarters Department of the Army. *Operations*, FM100-5. Washington, D.C., 14 June 1993.

Kelly III, Major Patrick, USA. "Modern Scientific Metaphors of Warfare: Updating the Doctrinal Paradigm." Monograph, School of Advanced Military Studies, U.S. Army Command and General Staff College, Fort Leavenworth, KS, Second Term Academic Year 92-93 [DTIC AD-A274 366].

Leader, Jeffery J. "Chaotic Keystream Generators for Additive Stream Ciphers." Technical Report NPS-MA-93-015. Monterey, California: Naval Postgraduate School, Jan 1993-April 1993 [DTIC AD-A265 824].

LeGare, Maj J. Marc USA. "Paradigm Found—The Nuclear and Nonlinear Battlefields." School of Advanced Military Studies, US Army Command and General Staff College, Fort Leavenworth, Kansas, First Term Academic Year 1992-93. February 1993 [DTIC AD-A264 451].

MacFarland, Maj Sean B., USA. "Non-Linear Operations: A New Doctrine for a New Era." School of Advanced Military Studies, US Army Command and General Staff College, Fort Leavenworth, Kansas, Second Term Academic Year 1993-94. 20 Apr 94 [DTIC AD-A284 137].

Mueller, Lt Col Theodore H., USA. "Chaos Theory: The Mayaguez Crisis." U.S. Army War College, Military Studies Program, Carlisle Barracks, Pennsylvania. 15 March 1990.

Oral, Zeki Okan. "Hopf Bifurcations in Path Control of Marine Vehicles." Thesis, Naval Postgraduate School, Monterey, California: June 1993 [DTIC AD-A268 818].

"Project 2025 Briefing for the Secretary of Defense and the Service Chiefs." Institute for National Strategic Studies, National Defense University, Washington, D.C., 1994.

Ray, William J. "EEG and Chaos: Description of Underlying Dynamics and Its Relation to Dissociative States." Final Progress Report: NASA Grant #NAG-1-1441, February, 1994 [NASA-CR-195171].

Sarigul-Klijn, Martinus M. "Application of Chaos Methods to Helicopter Vibration Reduction Using Higher Harmonic Control." Ph.D. dissertation, Naval Postgraduate School, Monterey, California: March 1990 [DTIC AD-A226 736].

Stright, Capt James R., USAF. "Embedded Chaotic Time Series: Applications in Prediction and Spatio-Temporal Classification." Ph.D. dissertation, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio: June 1994 [DTIC AD-A280 690].

Tagarev, Todor, Michael Dolgov, David Nicholls, Randal C. Franklin and Peter Axup. "Chaos in War: Is It Present and What Does It Mean?" Report to Air Command and Staff College, Maxwell Air Force Base, Alabama, Academic Year 1994 Research Program. June 1994.

Yim, Solomon C.S. "Chaotic and Random Responses of Ocean Structural Systems." Office of Naval Research Young Investigator Award, Research Project N00014-88-K-0729: 1 March 1992 [DTIC AD-A247 561].

Lectures, Interviews and Other Sources

Handel, Prof Michael I. "German Strategic Alternatives." Lecture at U.S. Naval War College, Newport, R.I.: 26 September 1994.

Interview with Dr John Hanley, Program Director of U.S. Naval War College Strategic Studies Group, and CDR Bill Millward, USN, Naval War College, Newport, RI. 16 December 1994.

Interview with Dr Stephen O. Fought, National Security Decision Making Department, Naval War College, Newport, R.I.: 1 February 1995.

Mayer-Kress, Gottfried. "Chaos and Crises in International Systems." Lecture at SHAPE Technology Symposium on Crisis Management, Mons, Belgium: 19-20 March 1992.

"NOVA: The Strange New Science of Chaos." PBS television broadcast NOVA #1603, 31 January 1989, produced by WGBH, Boston, Massachusetts.

Published descriptions of off-the-shelf products available from Iterated Systems, Inc., 5550-A Peachtree Parkway, Suite 650, Norcross, GA 30092.

Telephone conversations with Lt Col Gerald Diaz, USAF, Defense Airborne Reconnaissance Office, Washington D.C. November 1994.

Telephone conversations with Maj Bruce M. Deblois, School of Advanced Air Power Studies, Maxwell Air Force Base, Alabama. September 1994.

Telephone interview with Rick Darby, Iterated Systems, Inc., Norcross, Georgia. 28 November 1994.

Teller, Prof Edward. Comments at The Tenth Annual Space Symposium, Broadmoor Hotel, Colorado Springs, Colorado: February 1994.

Yorke, Jim. "Calculated Trajectories in Chaotic Systems." Lecture at Georgia Institute of Technology, School of Mathematics, Atlanta, Georgia: Fall 1989.